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COMPLEX CHARACTERIZATION OF TEXTILE SURFACE PROFILE

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Abstract. Thickness variation is one of important characteristics of textiles for special clothing acting as special barrier against influence of aggressive environment and industrial textiles as well. This characteristic is one of key parameters of quality of textiles production. The result of measurements is the so-called thickness profile. The proposed analysis of this profile is based on the spectral analysis, statistical analysis and calculation of effective fractal dimension. This scale invariant characteristic of thickness variation is suitable for characterization of complex curves. From spectral density function it is possible to estimate the typical periodicity caused by textile pattern. The application of these characteristics for description of special barrier clothing (heat resistant) surface thickness variation is discussed.

Резюме. Вариация толщины является одной из важнейших характеристик качества текстильных изделий испольуемых в промышленности. Она определяется по измерениям так называемого профиля толщины. В этой статье предложен метод анализа этого профиля на основе спектрального анализа, статистического анализа и вычисления эффективной фрактальной размерности.

1. Introduction

Roughness of engineering surfaces has been traditionally measured by the stylus profiling method creating surface profile [7]. This profile characterizes thickness (height) variation in selected direction. Modern methods are based on the image processing of surface images [10]. Surface irregularity of plain textiles has been identified by friction [1], contact blade [2,4], lateral air flow [3], step thickness meter [6] or subjective assessment [5].

Standard methods of surface profile evaluation are based on the relative variability characterized by the variation coefficient (analogy with evaluation of yarns mass unevenness) [8] or simply by the standard deviation. This approach is used in Shirley software for evaluation of results for step thickness meter [9]. Greenwood [4] proposed technique based on the definition of local maxims (peaks) and derivation of peaks height distribution. A lot of recent works is based on the assumption that the stochastic process (fractional Brownian motion) can describe thickness variation [13]. For characterization of smoothness is then possible to use Hausdorf or fractal dimension D [12]. Fractal dimension is closely connected with statistical characteristics of signals as variogram, correlogram and power spectral density function.

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This work is devoted to the analysis of thickness variation $R(d)$ obtained from Shirley step thickness meter. The combination of spectral analysis and fractal dimension estimation is used for evaluation of surface roughness of barrier technical textiles having various structures.

2. Surface profile characterization

Variation of thickness or surface roughness can be generally assumed as combination of random fluctuations (uneven threads, spacing between yarns, non uniformity of production etc.) and periodic fluctuations caused by the repeated patterns (twill, cord, rib etc.) created by weft and warp yarns. For description of roughness $R(d)$ as thickness of fabric in places $0 < d < T$ (T is maximum investigated sample length and M is number of places) the following group of characteristics have been proposed:

- Characteristics based on the surface roughness [11],
- Basic statistical characteristics of $R(d)$ [2],

Characteristics based on the measures of short range and long range dependencies (as fractal dimension [6]).

Due to complexity of $R(d)$ the fractal dimension often suitable as overall measure of roughness. Especially for weaves it is necessary to identify periodic component in $R(d)$ as well. For this purpose the spectral analysis can be useful. The position of repeated weave pattern can be estimated from variance spectrum (spectral density function) $S(d)$ estimated from periodogram. In this section the classical roughness parameters are discussed. Fractal dimension characteristics are the main ones used in this work.

2.1. Roughness parameters.

Let $R(d_j)$ represents the measurements of thickness at points $d_1, d_2, \ldots, d_M$. The measurement points d_j are often selected as equidistant and then $R(d_j)$ can be replaced by the variable R_j. For identification of positions in length scale is sufficient to know sampling distance $ds = d_j - d_{j-1} = T/M$ for $j > 1$. The standard roughness parameters used frequently in practice are [19]:

(i) **Mean Absolute Deviation (MAD)**. This parameter is equal to the mean absolute difference of surface heights from average value (R_a). For a surface profile this is given by

$$\mathrm{MAD} = \frac{1}{M} \sum_j |R_i - R_a| \,. \tag{1}$$

This parameter is often useful for quality control. However, it does not distinguish between profiles of different shapes. Its properties are known for the case when R_j's are independent identically distributed (i.i.d.) random variables

(ii) **Root Mean Square Value (RMS)**. This is given by

$$\mathrm{RMS} = \sqrt{\frac{1}{M} \sum_j (R_i - R_a)^2} \,. \tag{2}$$

Its properties are known for the case when R_j's are independent identically distributed (i.i.d.) random variables. One advantage of RMS over MAD is that for normally distributed data can be simple to derive confidence interval and to realize statistical tests. RMS is always higher than MAD and for normal data is RMS$=$ 1.25$*$MAD. It does not distinguish between profiles of different shapes as well. The parameter RMS

is less suitable than MAD for monitoring certain surfaces having large deviations (corresponding distribution has heavy tail).

(iii) **Mean Height of Peaks (MP)**. This is calculated as the average of the profile deviations above the reference value R (often is $R = R_a$). It is given as mean value of peaks P_i, $i = Np$ where

$$P_i = R_i - R \text{ for } R_i - R > 0 \text{ and } P_i = 0 \text{ elsewhere}.$$

(iv) **Mean Height of Valleys (MV)**. This is calculated as the average of the profile deviations below the reference value R (often is $R = R_a$.). It is given as mean value of valleys V_i, $i = Nv$ where

$$V_i = R - R_i \text{ for } R_i - R < 0 \text{ and } V_i = 0 \text{ elsewhere}.$$

The parameters MP and MV give information on the profile complexity. Exceptional peaks or valleys are not considered but are useful in tribological applications.

(v) **The Standard Deviation of Profile Slope (PS)**. This is given by

$$\text{PS} = \sqrt{\frac{1}{M}\sum_j \left(\frac{dR(x)}{dx}\right)_j^2}. \tag{3}$$

(vi) **The Standard Deviation of Profile Curvature (CP)**. This quantity called often as waviness is defined by the similar way

$$\text{PC} = \sqrt{\frac{1}{M}\sum_j \left(\frac{d^2R(x)}{dx^2}\right)_j^2}. \tag{4}$$

The slope and curvature are characteristics of a profile shape. The PS parameter is useful in tribological applications. The lower the slope the smaller will be the friction and wear. Also, the reflectance property of a surface increases in the case of small PS or PD.

(vii) **Mean Slope of the Profile (MS)**. This is given by

$$\text{MS} = \frac{1}{M}\sum_j \left|\frac{dR(x)}{dx}\right|_j. \tag{5}$$

Mean slope is an important parameter in several applications such as in the estimation of sliding friction and in the study of the reflectance of light from surfaces.

(viii) **Ten Point Average (TP)**. This characteristic is defined as the average difference between the five highest peaks and five deepest valleys within a surface profile. The parameter TP is sensitive to the presence of high peaks or deep scratches in the surface and is preferred for quality control purposes.

These parameters are useful in the case of functional surfaces or for characterizing surface bearing and fluid retention and other relevant properties. For the characterization of hand will be probably best to use waviness PC. The characteristics of slope and curvature can be computed for the case fractal surfaces from spectral density autocorrelation function or variogram.

3. Spectral analysis

The primary tool for evaluation of periodicities is expressing of signal $R(d)$ by the Fourier series of sine and cosine wave

$$R(d) = \frac{a_0}{2} + \sum_k (a_k \cos(\pi * k * d) + b_k \sin(2\pi * k * d)) \,. \tag{6}$$

Quantity d is often time or distance from origin and $k = 1, 2, 3, 4 \ldots$. The first two terms have period 1, the second two terms have period $1/2$, the third two terms have period $1/3$ etc. One consequence of this is that the different pairs of terms are orthogonal (integral of their product is zero). This fact facilitates fitting of Fourier series to experimental data. The term $a_0/2$ can be made zero by centralization (i.e. subtracting of mean value). By using of Euler formula $\exp(ia) = \cos(a) + i\sin(a)$, where i is imaginary unit the Fourier series may be written in the compact form

$$R(d) = \sum_k c_k \exp(-2\pi * k * d) \,. \tag{7}$$

The complex coefficients c_k have real and imaginary part a_k and ib_k In Fourier series only the terms up to $k = M/2$ contain any useful information. After this bound are real coefficients repeated symmetrically and imaginary coefficients repeated antisymmetrically. The Fourier Transform is conversion of data from series according to d to the series of frequencies $\omega = 2\pi * k/(M * T)$, for $k = 1, 2, 3 \ldots$

$$\mathrm{RF}(\omega) = \sum R(d) * \exp(-i * \omega * d) \,. \tag{8}$$

Function RF is symmetric about frequency $\omega = \pi/T$. For discrete data the fast Fourier Transform (FFT) leads to transformed complex vector DRF. Vector DRF may be used for creation of power spectral density. $S(\omega)$

$$S(\omega) = \mathrm{DRF} * \mathrm{conj}(\mathrm{DRF})/T^2 = \mathrm{abs}(\mathrm{DRF})^2/T^2 \tag{9}$$

where conj$(\cdot)$ denotes conjugate vector. The $S(\omega)$ is estimator of spectral density function and contains values corresponding to contribution of each frequency to the total variance of $R(d)$. Frequency of global maxim on $S(\omega)$ is corresponding to the length of repeated pattern and height corresponds to the nonuniformity of this pattern. Spectral density function is therefore generally useful for evaluation of hidden periodicities.

The estimation of the spectral density function $S(\omega)$ is relatively straightforward in theory but in practice situation is more difficult since data are only available in discrete samples of limited extent. For finite sample lengths if is necessary to use windowing (avoiding leakage) de-trending (avoiding non stationarity of mean) and filtration of parasite frequencies. Several methods of estimating the spectral density function are available. More precise estimates can be obtained by using of sophisticated procedures as averaged periodogram of overlapped windowed signals (Welch method) or multiple signal classification (MUSIC). The maximum entropy spectral analysis (MEM) provides smoother and higher resolution spectra for red-noise processes, which therefore would appear to be more suitable for good estimation. The method of MEM spectral estimation use of the Fourier transform between $S(\omega)$ and the autocorrelation function [16]. It is necessary to specify before computation the order of AR model. $S(\omega)$ is selected to maximize entropy such that the inverse Fourier transform of $S(\omega)$ yields the autocorrelation function. These spectral estimators

are available in Signal Processing toolbox of MATLAB system. For the white noise (independent standard normal random numbers) are the estimators of spectral density on the Fig. 1.

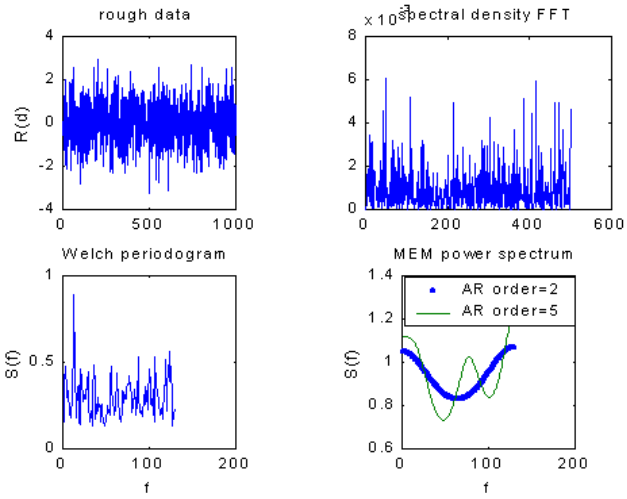

Fig. 1 *Raw data (white noise) and estimators of power spectral density.*

It is clear that the rough FFT based estimator shows the random fluctuations. Both more sophisticated estimators show the one or more periodicities.

For simulation behavior of these estimators for periodic structure with added random noise $N(0,1)$ the function

$$R(d) = 3 * \sin(2 * \pi * 10 * t) + 4 * \sin(2 * \pi * 4 * t) + N(0,1)$$

was generated. The estimators of spectral density and raw data are given on the Fig. 2.

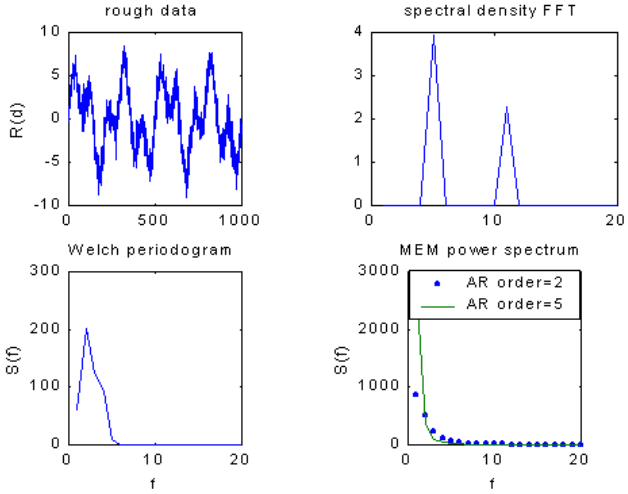

Fig. 2 *Spectral densities for periodic function with added whiter noise.*

For very high level of noise $N(0,400)$ are these estimators non effective see Fig. 3.

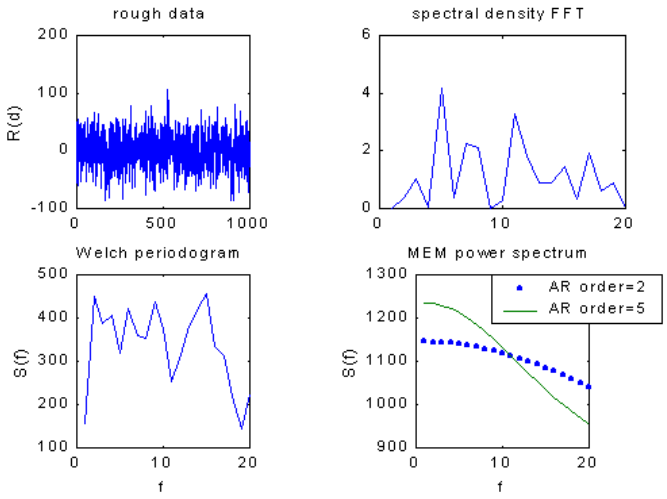

Fig. 3 *Spectral estimators for data from Fig. 2 with high level of noise* $N(0, 400)$.

The spectral estimators for finite data length and corrupted by random errors could be inaccurate. The more sophisticated procedures are very sensitive to the tuning parameters. For estimation of fractal dimension is therefore the best way to use simple FFT based method with proper data pretreatment (detrending, windowing).

4. Statistical analysis

A basic statistical feature of $R(d)$ is autocorrelation in distance. Autocorrelation depends on the lag h (i.e. selected distances between places of thickness evaluation). The main characteristics of autocorrelation is covariance function $C(h)$

$$C(h) = \mathrm{cov}(R(d), R(d+h)) = E(R(d) * (R(d+h) - E(R(d)))) \tag{10}$$

and autocorrelation function $\mathrm{ACF}(h)$ defined as

$$\mathrm{ACF} = \frac{C(h)}{C(0)} \tag{11}$$

ACF is one of main characteristics for detection of short and long-range dependencies in time series. It could be used for preliminary inspection of data. The computation of sample autocorrelation directly from definition is for large data tedious. The technique of ACF creation based on the FFT (there is a known transformation of ACF to $S(\omega)$ and back) is contained in Signal Processing toolbox of MATLAB (procedure xcorr.m).

In spatial statistics is more frequent variogram, (called often as structure function) which is defined as one half variance of differences $(R(d) - R(d+h))$

$$\Gamma(h) = 0.5 * D[R(d) - R(d+h)] \tag{12}$$

or

$$\Gamma(h) = 0.5 * \left[E(R(d) - R(d+h))^2 - (E(R(d) - R(d+h)))^2\right]. \tag{13}$$

For stationary random process is mean value independent on lag h i.e. $E(R(h)) = m$ and then

$$\Gamma(h) = 0.5 * E(R(d) - R(d+h))^2 \,. \tag{14}$$

For random processes having stationarity of second order is valid

$$C(h) = E[R(d) * R(d) * R(d+h)] - m^2 \,. \tag{15}$$

Variance is then equal

$$D(R(d)) = C(h=0) = C(0) \tag{16}$$

and variogram is directly related to covariance

$$\Gamma(h) = C(0) - C(h) \tag{17}$$

The variogram is relatively simpler to calculate and assumes a weaker model of statistical stationarity„ than the power spectrum. Several estimators have been suggested for the variogram. The traditional estimator is

$$G(h) = \frac{1}{2N(h)} \sum_{j=1}^{N(h)} (R(d_j) - R(d_j + h))^2 \tag{18}$$

where $N(h)$ is the number of pairs of observations separated by lag h. Problems of bias in this estimate when the stationarity hypothesis becomes locally invalid have led to the proposal of more robust estimators. One such estimator has been created by Cressie and Hawkins [14]. Another estimator has been suggested by Isaaks and Srivastava [15]. This makes use of the estimated covariance function to obtain the non-ergodic variogram, which is also referred to as the inverted covariance defined as

$$Gd(h) = D(R(d)) - \frac{1}{2N(h)} \sum_{j=1}^{N(h)} (R(d_j) - m_d(h)) * (R(d_j + h) - m_d(-h)) \tag{19}$$

where $D(R(d))$ is replaced by the sample variance, and are estimated lag means for the head and tail of the data pairs vector. Both variogram and ACF can be used for estimation of fractal dimension estimation.

For the white noise (independent standard normal random numbers) are the estimators of ACF, variogram and covariance function on the Fig. 4.

The same estimators are given for the case of periodic function with added white noise (see Fig. 2) on the Fig. 5.

For very high level of noise $N(0, 400)$ are periodicities hidden are not detected by these estimators. This level of noise is not in practice result of measurement errors but random fluctuations of surface profile.

It is can be summarized that simple statistical characteristics are able to identify the periodicities in data but the reconstruction of “clean” dependence is more complicated. The variogram is often sufficient for characterization of surface profiles.

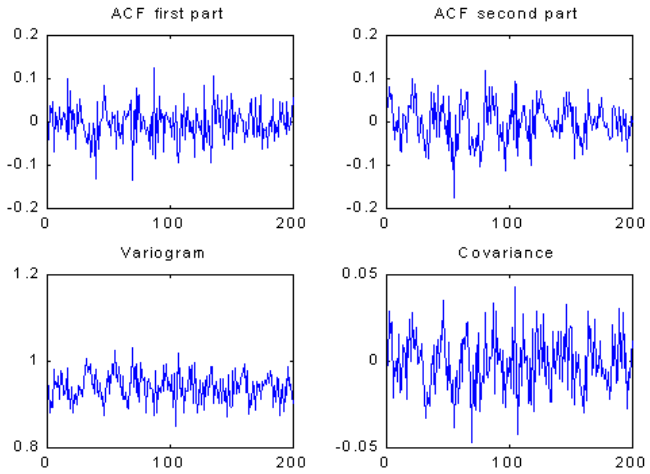

Fig. 4 *Estimators of autocorrelation function, variogram and covariance function for white noise.*

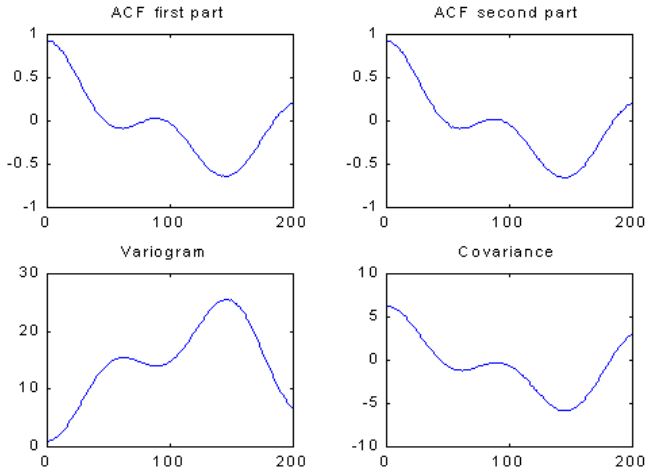

Fig. 5 *The ACF, variogram and covariance function for periodic function with added white noise.*

5. Chaos dynamics

This approach is common for the time series, which can be assumed to be result of dynamic process. Surface profiles are often results of deterministic process embedded in noise (as surface of weaves). For some surfaces the deterministic chaos dynamic can be accepted (formation of some types of nonwovens etc.) The key to modeling is that even if the exact description of the dynamic system under study is unknown, the state space can be reconstructed from a single scalar surface profile. When the state space is reconstructed from a scalar data serie it is usual to call this state space a phase space. The phase space is defined as the multidimensional space whose axes consist of variables of a dynamic system. The dimension of the attractor will provide

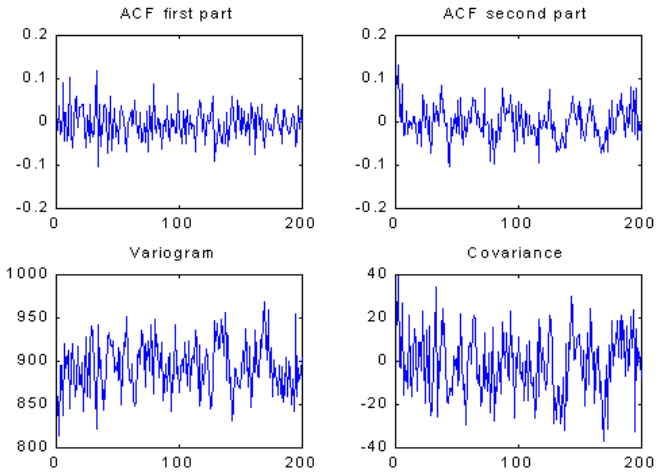

Fig. 6 *ACF, variogram and covariance for periodic data embedded in high level noise* $N(0, 400)$.

a measure of the minimum number of independent variables that describe the dynamic system.

The state space reconstruction is the basis for recovering the properties of the original attractor from a scalar data series. Therefore, building a dynamic model here involves reconstructing a phase space from the data.

A method of time delay coordinate has been suggested for this purpose. Takens [20] showed that an attractor, which is topologically equivalent to the scalar time series, could be reconstructed from a dynamic system of n variables by using the time-delay coordinates. Specifically, given scalar data series $i = 1, \ldots, M$ the m dimensional signal $\mathbf{X}i$ is composed of the scalar series R_i as follows:

$$X_i = R_i, Ri_{i+\tau}, R_{i+2\tau}, \ldots, R_{i+(m-1)\tau}\,.$$

The τ is an appropriate length delay and m is an embedding dimension. In the process of constructing a well-behaved phase space an important question is how to choose the delay (τ) and the embedding dimension (m). Usually, the procedure of determining the embedding dimension m is to increase m and to estimate the fractal dimension or the largest Lyapunov exponent for every embedding dimension until the fractal dimension or the largest Lyapunov exponent remains almost constant. The delay, τ is often chosen from the autocorrelation function of the original time series as the delay τ at which the autocorrelation function attains the value of $1/e$. A justification for the above procedure in the deterministic case is given by Takens theorem [20]. According of this theorem is $m = 2D + 1$ where D is the attractor dimension (in the sense of Hausdorff dimension). In practice, D is unknown. An estimate of the attractor dimension, D, may be obtained from the correlation dimension.

A correlation dimension, which provides the number of active degrees of freedom of the system, can be constructed in phase space as:

$$d_2 = \lim_{r\to 0} \lim_{M\to\infty} \frac{\ln C^m(r)}{\ln r} \tag{20}$$

where

$$C^m(r) = \frac{1}{L(L-1)} \sum_{i,j=1}^{L} H\left(r - \|X_i - X_j\|\right) . \tag{21}$$

The $H(u)$ is Heaviside step function ($H(u) = 1$, if $u \geq 0$ and $H(u) = 0$ for $u < 0$), $\|X_i - X_j\|$ is the norm computed between two vectors and $L = M - (m-1)\pi$, with M being the number of data points. The quantity $C^m(r)$ for $M \to \infty$ is called the **correlation integral**. When the system dynamics is governed by a strange attractor, it is possible to show that for a sufficiently small value of r

$$C^m(r) \approx r^{D(m)} \qquad \text{and} \qquad \lim_{m\to\infty} D(m) = d_2 \tag{22}$$

Thus, if the value of d_2 stabilizes at some value $d*_2$, as the embedding dimension increases, then $d*_2$ is the estimated correlation dimension A minimal data length required for good estimation of the dimension of the attractor is $n > 10^{d*_2/2}$.

Correlation dimension is one of the techniques used in detecting the existence of chaos. This technique has some limitations:

i The need for a large amount of scalar time series data.
ii The crucial influence of noise on its computation.
iii If the attractor has an integer correlation dimension, then it is not clear whether the process is chaotic.

Thus, to detect the existence of chaos the correlation dimension should be complemented by other techniques, such as the Kolmogorov K-entropy and the Lyapunov exponent.

A useful quantity, which can be extracted from the correlation integral, is the Kolmogorov K-entropy.

$$K_2 \approx \lim_{m\to\infty} \lim_{r\to 0} K_2^m(r) \;\text{ where }\; K_2^m(r) = \frac{1}{r} \log \frac{C^m(r)}{c^{m+1}(r)} . \tag{23}$$

If the value of K_2^m stabilizes at some value K_2, as m increases, that K_2 is the entropy estimate. The K_2 entropy measures the degree of chaos in a system:

- Regular or ordered systems are characterized by $K_2 = 0$.
- Purely random systems are characterized by $K \to \infty$.
- Chaotic (deterministic) systems are characterized by $0 < K < +\infty$.

Additional features that describe deterministic processes give Lyapunov exponents. These exponents measure the exponential divergence or convergence of nearby orbits. Points close together in phase-space are nearly identical states; points with separating orbits become unaligned with each other. There are n Lyapunov exponents in a given n-dimensional system, but we need to estimate only the largest one, namely $\gamma_{\max}$, which is defined by

$$\gamma_{\max} = \lim_{n\to\infty} \frac{1}{n\gamma} \sum_{i=1}^{n} \log \|\varepsilon(\gamma * i)\| \tag{24}$$

where $\varepsilon(\gamma i)$ is the difference of infinitesimal neighboring orbits on the attractor. If the dimension of the attractor is not an integer, or if there is at least one positive Lyapunov exponent, the system is said to be chaotic. However, by applying only the largest positive Lyapunov exponent one cannot distinguish some nonlinear models from chaotic models. In addition, the finiteness of the correlation and the information

dimension as well as the largest positive Lyapunov exponent do not imply that one can definitely distinguish between a random process and a chaotic deterministic process. it should be recognized that these invariants are not statistical tests.

One test of particular interest here is the theorem by Brock on the dynamic properties of the residuals from the best fitting data series model. Brock showed that if data series are chaotic, the estimated correlation dimension and the largest Lyapunov exponent of residuals from the best fitting time series model are the same as that of the original data. The test statistic is given by

$$W = \sqrt{L} * \left[C^m(r) - (C^1(r))^2\right] / \sigma_m(r) \tag{25}$$

where $C^m(r)$ and $C^1(r)$ are given in Eq. (20) and $\sigma_m(\gamma)$ is an estimate of the standard deviation. Under the null hypothesis of an independent identical distribution (white noise), the W statistic converges as $M \to \infty$ to a standard normal variable with mean zero and variance one. A large value of W is evidence of a non-linear model and W value of 0 indicates a stochastic process. Based on Monte Carlo results, it can be suggested that the W statistic could detect other types of departures from i.i.d. such as non-linearity, non-stationarity and deterministic chaos. Therefore, rejection of i.i.d. does not imply chaos. The null hypothesis of i.i.d. may be rejected because of linear or non-linear dependence or chaotic structure.

6. Surface profiles inspection

Given a surface profile, the selection of the appropriate approach for its analysis is not a trivial task because the mathematical background of the underlying process is unknown. Moreover, the surface profiles are corrupted by noise and consist of finite number of sample values. Many surface profiles from real nature recorded precisely enough have similar properties, and thus the task for their description could be generalized in standard procedures. The task to analyze real data is often to resolve the so-called inverse problem, i.e., given a surface profile, how to discover the characteristics of the underlying process

Three approaches are mainly applied: one based on random stationary processes, the second based on the self affine processes with multiscale nature and the third based on the theory of non-linear deterministic systems (chaotic dynamics). The majority of the important results in processing and analysis of surface profiles have been obtained considering the signals consisting of multiperiodic components mixed with random noise.

The problem becomes more complicated if one wishes to determine whether the surface profile under study is governed by stochastic or chaotic process (as a whole), as the surface profile change their structure and dynamics over time. Hence, methods for tracking the dynamic behavior should suit better.

Before choosing the approach, some preliminary analysis is needed mainly to test the signals for non-stationarity and non-linearity. This is important as some kind of stochastic processes with power-law shape (self affine surfaces) of their spectrum may erroneously be classified as chaotic processes on the basis of some properties of their non-linear characteristics, e.g., correlation dimension and Kolmogorov entropy. In this sense, the tests for non-stationarity and non-linearity may be regarded as a necessary preprocessing in order to choose an appropriate approach for further analysis.

Prior to selecting any method for data analysis, some simple tests are useful to apply on the data surface profile [18]. The first one may be to observe the amplitude

distribution. In most of the methods for data processing based on stochastic models, gaussian distribution is assumed. If the distribution is proved to be non-gaussian (according to some test or inspection), there are three possibilities:

(1) the process i s linear but non-gaussian;
(2) the process has linear dynamics, but the observations are as a result of non-linear "static" transformation (e.g. square root of the current values) [18];
(3) the process has non-linear dynamics.

It is suitable to construct the amplitude $R(d_i)$ histograms for the first and second half of data separately and inspect non-gaussianity or asymmetry of distribution.

The second test is based on inspection of the shape of the autocorrelation function (ACF). In case of a proper selection of Nyquist sampling frequency, the slow decrease of ACF for large lags indicates long-range correlation, which may be due to non-stationarity and/or dynamic non-linearity [17]. In case of non-stationarity due to additive (e.g. trend-like) components, they could be removed in order to test the ACF on the residuals. If the shape ACF remains the same, this is a typical case of $1/\omega^{\beta}$ noise ($0 < \beta 2$), which suggests fractal dynamics. If $\beta > 1$, the process is like Brown noise and possesses intrinsic non-stationarity. The mean and variance are not defined as they depend on the time scale, and there is no sense in collecting more data samples.

The presence of non-linearity may be tested by the comparison of the shape of square of ACF with that of ACF of squares of samples. If $\mathrm{ACF}(R_i^2) > (\mathrm{ACF}(R_i))^2$, non-linear process is suggested. More sophisticated tests are given in [18].

7. Nature of surface profiles

Most of man made objects are geometrically simple and can be classified as composition of regular geometric shapes as lines, curves, planes, circles, spheres etc. Some objects are not be approximated precisely by the regular geometric shapes. One category of these objects is called **fractals**. Benoit Mandelbrot has coined term fractal in the seventies. (From Latin **fractus**, meaning irregular or fragmented.) Fractals have two interesting characteristics. First of all, fractals are **self-similar** on multiple scales, in that a small portion of a fractal will often look similar to whole object. Second, fractals have a **fractional dimension**, as opposite to integer dimension of regular geometrical objects. Because fractals are self similar they are constructed by recursion. For **geometrical fractals** is the recursion explicitly visible. Typical example is so called Koch curve shown on Fig. 7.

The interesting facet of the Koch curve is its length LK. In the n-th step is length of segment equal to LS$= 1/3^n$ and the curve consists of 4^n segments. Therefore it is LK$= (4/3)^n$.

For **stochastic fractals** or **random fractals** the recursion is more little subtle and may be an artifact of an underlying fractal building process that occurs on multiple spatial scales. The typical generating function is Weistrass Mandelbrot equation, which satisfies to the self-affinity requirement (replaces of self similarity for case of functions). The height $R(d)$ of surface in the point d i.e. surface profile is equal to

$$R(d) = G^{D-1} \sum_{i=n_1}^{\infty} \frac{\cos(2\pi g^i d)}{g^{i*(2-D)}} \tag{26}$$

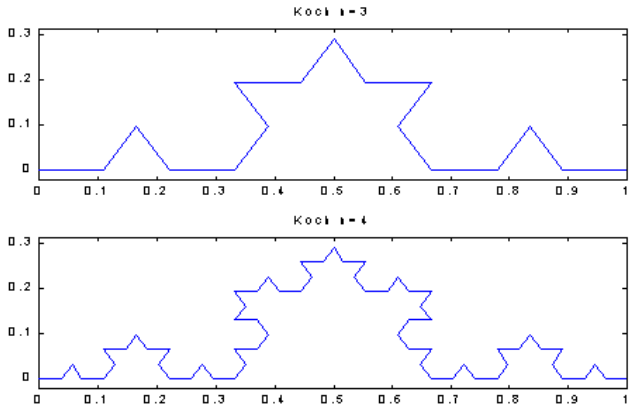

Fig. 7 *Koch curve (in each step of construction the middle portion of each segment is removed and replaced by two new line segments. First step is line, having length 1).*

where $1 < D < 2$ is fractal dimension, G is characteristic length scale of surface and g^i determines the frequency spectrum of surface roughness. The suitable value of this parameter is $g = 1.5$. The simulated stochastic fractals generated by eqn. (26) for $D = 0.75$ and $D = 0.75$ are shown on Fig. 8. On the same figures are power spectral densities in log-log scale, variograms in log-log scale and amplitude histogram. The practically perfect linearity of log vartiograms and scattered linearity of log power spectral densities are typical for self-affine curves. Histograms of amplitudes R_i indicate multimodality or skewed distribution.

These curves correspond to the nonstationary random process and describe fractional Brownian motion fB. The lowest frequency is then related to the sample length L according to relation $g^{n_l} = 1/L$.

The evaluation of D and G from random fractals is based on the power spectral density $P(\omega)$ function, which has for eqn. (26) the power law form

$$(27)\quad P(\omega) = C * \omega^{-\beta} \text{ where } C = \frac{G^{2(D-1)}}{2ln(g)} \text{ and } B = 5 - 2D \text{ for } l/L \leq \omega \leq \infty\,.$$

The same power law behavior (differences are in the sign of power) is valid for variogram.

The power law form is typical feature of fractals.

In some cases are fractals stationary random processes like fractional Gaussian noise fG. The stationary fG process can be simply obtained as successive differences of fB process. Stationary fG process corresponding to fB from Fig. 8. is shown on the Fig. 9.

Fractals in the form of fG are fully characterized by fractal dimension D and variance σ^2.

A lot of engineering surfaces obey fractal like behavior for high frequencies. For small frequencies the log-log power spectral density exhibits nearly constant portion. The Markov type models can express this behavior. Simplest one has form

$$(28)\qquad R_{i-1} = r * R_i + (1 - r) * \sigma * N(0, 1)$$

where r is autocorrelation coefficient. This type of profile is shown on the Fig. 10.

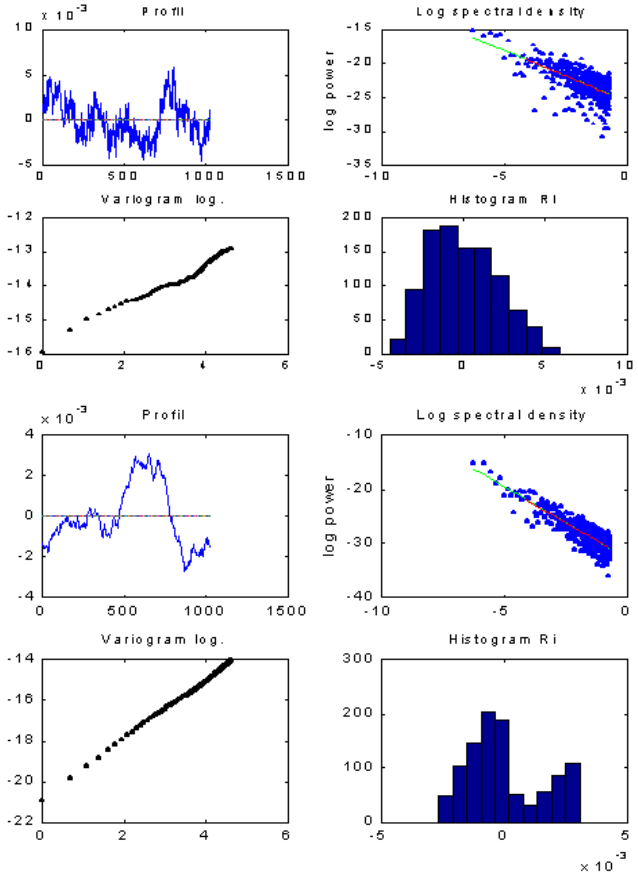

Fig. 8 *Stochastic fractals generated by eqn. (26) for* $D = 0.25$ *and* $D = 0.75$.

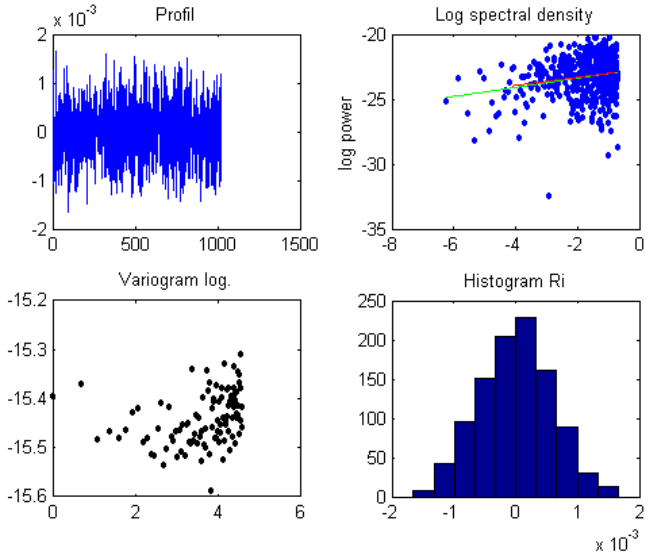

Fig. 9 *Fractional Gauss noise corresponding to first difference of the fractional Brownian motion from Fig. 8.*

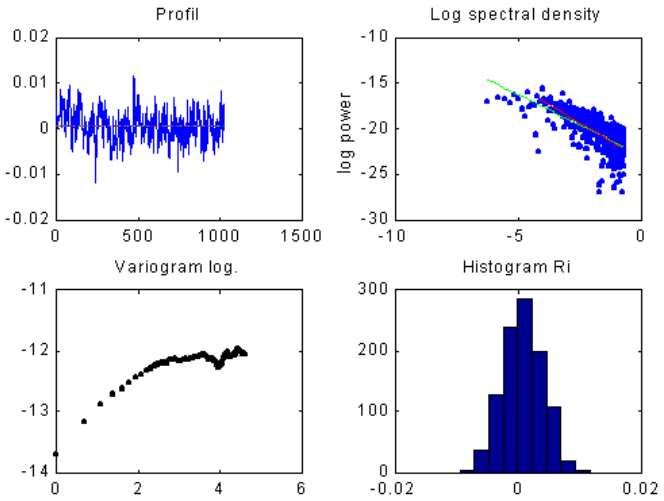

Fig. 10 *Markov type surface profile generated by the eqn (28).*

It is evident that the linearity for small lengths (corresponds to the high frequencies) is still very high for log-log variogram. Based on these and previous simulations we can conclude that:

- Power spectral density of Gauss noise has a lot of local extremes. Variogram has random fluctuations in small scale.
- Power spectral density of composite sine waves embedded in high-level noise exhibit no right pattern. The same is valid for variogram.
- Power spectral density of fractal type (fBm) surface profiles exhibits scattered linear trend according to the theory. The variogram exhibits more strict linearity with relative small scatter. Variogram is here typical power function of h.
- Fractional Gaussian surface profiles are typical by random fluctuations of variogram and symmetrical shape of amplitudes histogram.
- Markov like surface profiles have linear portion on variogram at smaller lags h. For higher lags the plateau is visible.

The very simply calculated variogram can replace the spectral power density. Only for periodic surface profiles with small noise level enables power spectral density the identification of periodicities.

8. Estimation of the fractal dimension

A convenient way of characterizing the smoothness of an isotropic surfaces is Hausdorf or fractal dimension. If the surface is very smooth is fractal dimension equal to $D_p = 2$. For extremely rough surfaces is fractal dimension approaching to limit value $D_p = 3$. General definition of fractal dimension is based on the capacity principle [12] In measurement of surface profile (thickness variation $R(h)$) the data are available through one dimensional line transect surface. Such data represents curve in plane. Two dimensional fractal dimension D *is then number between 1* (for smooth curve) and 2 (for rough curve). If a surface may be modeled by a stationary, isotropic Gaussian field then the relation

$$Dp = D + 1\,. \tag{29}$$

The expected variance of the increment of Brownian motion can be expressed using a value of the Hurst exponent H, where $H = 0.5$ [17]

$$E(R(d) - R(d+h))^2 \approx |h|^{2H}\,. \tag{30}$$

For fractional Brownian motion is H in the interval $(0, 1)$. Where $H = 0$ this denotes a surface of extreme irregularity and $H = 1$ denotes a smooth surface. Exponents H and fractal dimension D are in fact related

$$D = D\gamma + 1 - H \tag{31}$$

where $D\gamma$ is the topological dimension such that D is in interval $(2, 3)$ for a surface and $(1, 2)$ for a cut across a surface. Note that fractional Brownian motion can be expressed in terms of a power law variogram

$$\Gamma(h) \approx c|h|^H \tag{32}$$

where c is a constant. Similarly, for $S(d)$ is valid

$$S(d) = c_1 * |d|^{-(1+2H)} \tag{33}$$

where exponent $(1+2H)$ lies in the interval $(1, 3)$. Fractal dimension is conventionally obtained through estimating the parameter from a LSE linear regression of the log-log transformation of Equations (32) and (33). The same results can be obtained assuming that thickness variation $R(d)$ is stationary Gaussian process and covariance function $C(h)$ is sufficiently smooth [12,13]. The behavior of this function near the origin can be described by power type model

$$C(0) - C(h) \approx x|h|^{\alpha}\,. \tag{34}$$

Another possibility is to use spectral density

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} C(h) * \cos(\omega * t) dt\,. \tag{35}$$

In a neighborhood of infinity is spectral density expressed as power function

$$S(\omega) = c_1 * |\omega|^{-(\alpha+1)}\,. \tag{36}$$

Constant c_1 is dependent on the c and α only. Fractal dimension is then equal to

$$D = 2 - 0.5^{*}\alpha\,. \tag{37}$$

This formula may also be verified for another processes related to Gaussian one. In general it is D computed from this relation denoted as effective fractal dimension.

There are several problems with estimating fractal dimension in this fashion. First, elevation points, points on the variogram and the error term in the LSE regression are likely to be autocorrelated. Second, data points in log-log space are unequally spaced and, third, decisions concerning an acceptable cutoff for goodness of fit (R^2)

of the linear function are of an arbitrary a priori nature. Since the aim of the line fitting exercise in estimating fractal dimension is the description of the relationship rather than prediction, the bias introduced by the first problem is not critical. A solution to the second is to re-sample the data using a geometric progression, but at a cost of a dramatic reduction in the number of points used in the line fitting exercise. An alternative to the third is to estimate the standard error SE around the slope of a regression line. Based on these equations the program FRACT1 in MATLAB for estimation of fractal dimension from variogram and power spectral density has been constructed. Based on the preliminary testing the results of computation from variogram were more stable and reliable. From power spectral density the d corresponding to the global maximum of $S(d)$ can be evaluated as well

9. Experimental part

Typical weave for barrier (heat resistant) textiles has been selected. Pattern is **Satin** - warp 100% cotton, weft cotton/PET 65/35. Finishing Texaflam PSE/FRE.

The thickness variation curve $R(d)$ ($M = 450$ points) was evaluated by step thickness meter Shirley. The measuring plate has been replaced by the narrow blade. Frequency of sampling was 10 [1/min] and compression load was 2500 Pa. Main characteristics and fractal dimensions are summarized in the Table 1. Computations were realized by program FRAC1 in MATLAB.

Sample	I Satin
Mean [mm]	0.804
Variation coefficient [%]	2.41
Fractal dimension variogram – full data	1.552
Fractal dimension variogram – initial data	1.598
Fractal dimension power spectrum – full data	1.6024
Fractal dimension power spectrum – final data	1.585
Mean surface height Ra	0.8042
MAD	0.0167
RMS	0.0192
d for extreme on power spectrum	3

Tab. 1 *Basic characteristics for* $R(d)$.

Thickness variation $R(d)$, log-log power spectral density and variogram and histogram of amplitudes are shown on Fig. 11.

The comparison of $\mathrm{ACF}(R - R_a)^2$ and $\mathrm{ACF}((R - R_a)^2)$ for satin is given on Fig. 12.

There is detected of the some kind of no linearity.

10. Discussion

The characterization of thickness variation by combination of variogram and fractal dimension enables to characterize complex surface profiles. From table 1 and results from other tested fabrics the following conclusion can be created:

- Fractal dimension is not directly proportional to overall variability characterized by RMS or MAD.
- Fractal dimension characterize mainly the complexity of thickness variation.

- Values of maximum on the power spectral density d approximately agree with the dimensions of weave unit.
- Variation of fractal dimension computed form variograms enables classification of weaves.

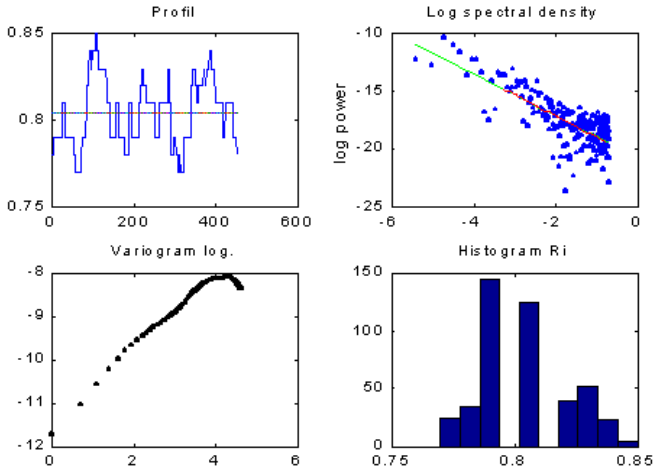

Fig. 11 *Thickness variation for sample I (satin).*

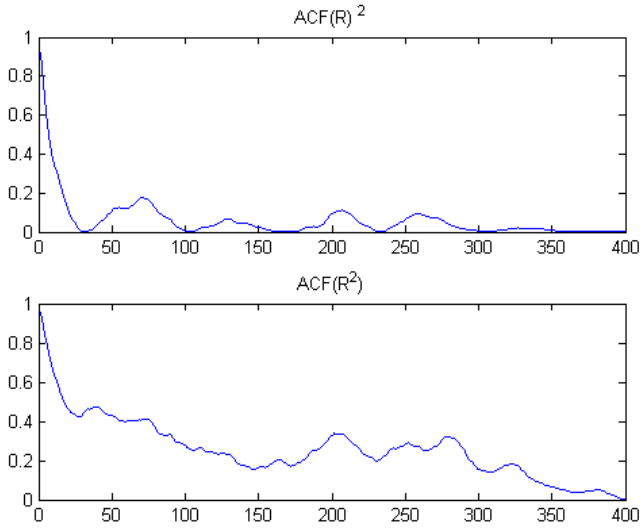

Fig. 12 *Comparison of $ACF(R)^2$ and $ACF(R^2)$ – centered data for satin.*

11. Conclusion

The application of variogram and power spectral density for evaluation of fractal dimension and characterization of periodic fluctuations of weave thickness has been shown. These characteristics can be used for deeper investigation of results from SHIRLEY step thickness meter. For more precise analysis should be estimator of power spectral density replaced by the more robust one or data should be properly filtered.

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