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RANK TESTS FOR A CHANGE IN CENSORED DATA

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ABSTRACT. A class of rank statistics for testing a change in distribution of randomly censored data developed by Hušková and Neuhaus (2001) is presented. The limit behaviour of these test statistics under null hypothesis of 'no-change' in distribution of censored variables is described. Particularly, the Koziol–Green model of random censorship is studied. The critical values for tests for a change in location model are obtained through permutation principle. Theoretical results are accompanied by simulation study.

Резюме. Рассмотрен класс ранковых статистик для проверки гипотезы о разладке по случайно цензурированным данным, предложенный Хушковой и Нейгауссом (2001). Изучены асимптотические свойства тестовых статистик при нулевой гипотезе. В частности, исследована статистическая модељ Козиола–Грина. Критические значения были получены методом перестановок. Теоретические резуљтаты иллюстрируются компьютерной симуляцией.

1. Introduction

Typically, in survival analysis and reliability theory the quantities of interest are only partially observable. The data can be *randomly censored*, *truncated* or *interval* censored, see [9], e.g. In this paper we consider the classical model of random censorship.

The ith subject[1] has nonnegative, independent latent *survival* and *censoring* times X_i^0 and C_i, e.g. the variable X_i^0 is the event of interest of life of the i-th patient included in the study. The patient can be withdrawn from the study due to many reasons, e.g. an accidental death or a migration of human population.

Actually, instead of survival times we observe pairs

$$X_i = \min(X_i^0, C_i), \quad \Delta_i = I\{X_i^0 \le C_i\}, \quad i = 1, \dots, n,$$

where the symbol $I(A)$ denotes the indicator of a set A.

We assume that $X_1^0, \dots, X_n^0$ and $C_1, \dots, C_n$ are mutually independent random variables such that for some unknown $\gamma \in (0,1]$ and $\eta \in (0,1]$ (η and γ need not be the same) $X_1^0, \dots, X_{[\gamma n]}^0$ and $X_{[\gamma n]+1}^0, \dots, X_n^0$ have the continuous distribution functions F_1 and F_2, respectively, $F_1 \ne F_2$, and $C_1, \dots, C_{[\eta n]}$ and $C_{[\eta n]+1}, \dots, C_n$ have the continuous distribution functions G_1 and G_2, respectively, $G_1 \ne G_2$. The point γ (or $[\gamma n]$) is called *the change point.*

We are interested in testing problem (H_0, A), where

$$H_0 : \gamma = 1 \quad \text{and} \quad A : \gamma \in (0,1). \tag{1.1}$$

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[1] the index i corresponds to chronological order the subject has entered into the study

This is one of the basic tasks in the change point analysis. More information about the change point analysis can be found in [3], e.g. The testing problem (1.1) does not concern censoring variables. Generally, we assume that the distribution of censoring variables can have changed. The change-point problem for censored data is considered only in a few papers, e.g. test procedures based on quantile function were developed in [1] or tests and an estimator of the change point based on U-statistics were studied in [10] and [5]. Particularly, limit properties of nonparametric tests for such a problem can be found in [4] and [7], e.g.

Remark. If the possible change point γ is known then the testing problem reduces to a two-sample testing problem with censored data for which a number of test statistics have been developed, see [8] and [9], e.g.

2. Test statistic and limit theorem

Hušková and Neuhaus in [7] have developed their test along the lines of a two-sample rank test for the random censorship applying *the union-intersection principle*, for more details see [3], e.g. The test procedure is based on

$$L_k = L_k(\tau_0) = \frac{|W_k(\tau_0)|}{\sqrt{V_k(\tau_0)}}, \quad k = 1, \dots, n-1, \tag{2.1}$$

where

$$W_k(\tau_0) = \frac{1}{\sqrt{n}} \sum_{j=1}^{k} a_n(j),$$

$$V_k(\tau_0) = \frac{1}{n} \int_0^{\tau_0} w_n^2(t) \, \frac{\sum_{j=1}^k Y_j(t) \sum_{j=k+1}^n Y_j(t)}{Y^2(t)} \, \mathrm{d}N(t) + v_k, \tag{2.2}$$

with

$$a_n(j) = \int_0^{\tau_0} w_n(t) \, \mathrm{d}N_j(t) - \int_0^{\tau_0} \frac{w_n(t) Y_j(t)}{Y(t)} \, \mathrm{d}N(t), \tag{2.3}$$

$$N_j(t) = \Delta_j I(X_j \le t), \qquad Y_j(t) = I(X_j \ge t);$$

$$N(t) = \sum_{j=1}^{n} N_j(t), \qquad Y(t) = \sum_{j=1}^{n} Y_j(t).$$

$$v_k = \frac{k(n-k)}{n^2} \left(I(k \le \log\log n) + I(k \ge n - \log\log n) \right)$$

Due to the terms v_k we ensure that $V_k(\tau_0)$ are bounded away from 0. The value τ_0 is such a positive number for which

$$0 < \tau_0 < \tau := \sup\{x;\ (1 - F_1(x))(1 - G_j(x)) > 0,\ j = 1, 2\}.$$

Appropriate choice or estimator of τ_0 has not yet been found.

The weight functions $w_n(X_j, \triangle_j; j = 1, \dots, n) \ge 0$ fulfil, as $n \to \infty$,

$$\sup_{0 \le t \le \tau_0} |w_n(t) - w(t)| = o_{\mathrm{P}}((\log\log n)^{-1}), \tag{2.4}$$

where w is a continuous nonrandom function on $[0, \tau_0]$. An important class of weight functions fulfilling (2.4) is $w_n(t) = (\hat{S}_n(t-))^{\rho} \left(\frac{Y(t)}{n}\right)^{\kappa} I(Y(t) > 0)$, where $\rho, \kappa \ge 0$ and $\hat{S}_n(t-) = \prod_{i: X_i < t} \left(1 - \frac{\Delta_i}{Y(X_i)}\right)$ is the left-continuous Kaplan–Meier estimate of the survival function.

Under the hypothesis H_0 and under mild assumptions the test statistic L_k defined in (2.1) has asymptotically standard normal distribution as $\min(k, n-k) \to \infty$, $k = 1, 2, \ldots, n-1$, see [8].

Since in our testing problem (1.1) the alternative is $A = \cup_{k=1}^{n-1} H_k$, $H_k : [\gamma n] = k$, we reject H_0 if at least one of L_k, $k = 1, \ldots, n-1$, takes large value. This leads to the rejection region

$$T_n = T_n(\tau_0) = \max_{1 \le k < n} L_k \ge c_n(\alpha),$$

where $c_n(\alpha)$ is determined in such a way that the test has the prescribed level α.

To apply this test procedure we need at least an approximation for the critical value $c_n(\alpha)$. In the change point analysis we usually get it through the limit distribution of the test statistic under H_0.

Theorem 2.1. *Suppose $X_1^0, \ldots, X_n^0$, $C_1, \ldots, C_n$ are independent random variables. Let $X_1^0, \ldots, X_n^0$ have arbitrary continuous distribution function F. Let $C_1, \ldots, C_{[n\eta]}$ have continuous distribution function G_1 and $C_{[n\eta]+1}, \ldots, C_n$ have continuous distribution function G_2 for some $\eta \in (0, 1]$. Let (2.4) be satisfied and let*

$$\int_0^{\tau_0} w(t)(1 - G_i(t))\,\mathrm{d}F(t) > 0, \quad i = 1, 2. \tag{2.5}$$

If $n \to \infty$, then for all $y \in \mathbb{R}$ we have

$$\mathrm{P}\Big(d_1(\log n)\, T_n(\tau_0) \le y + d_2(\log n)\Big) \to \exp\big\{ -2\mathrm{e}^{-y}\big\}, \tag{2.6}$$

where

$$d_1(t) = \sqrt{2\log t}, \qquad d_2(t) = 2\log t + \tfrac{1}{2}\log\log t - \tfrac{1}{2}\log \pi. \tag{2.7}$$

Proof. The proof can be found in [7]. □

3. Koziol–Green model

Consider *the Koziol–Green (KG) model* of random censorship, i.e., let us assume the relation between the distribution functions of survival and censoring variables in the form $1 - G(t) = (1 - F(t))^\lambda$ for some unknown constant $\lambda > 0$ and all t.

Remark. In the KG model $\gamma = \eta$, so under the hypothesis H_0 censoring variables $C_1, \ldots, C_n$ are i.i.d. too, and under the alternative A the distributions of survival and censoring variables have changed in the same time point. If $\lambda = 0$, the survival variables X_i^0's are not censored.

In the KG model the limit behaviour of $T_n(\tau_0)$ under H_0 can be obtained using *permutation principle.* In this case we take a random permutation $\mathcal{R} = (R_1, \ldots, R_n)$ of $(1, \ldots, n)$ and by Lemma 2.2 (special case $\eta = 1$) in [7] instead of $\frac{1}{\sqrt{V_k(\tau_0)}}$, where $V_k(\tau_0)$ is defined in (2.2), we can use the standardization $\sqrt{\frac{n^2}{k(n-k)}}\frac{1}{\sigma_n}$, where $\sigma_n^2 = \frac{1}{n-1}\sum_{j=1}^n (a_n(j) - \bar{a}_n)^2$ and $\bar{a}_n = \frac{1}{n}\sum_{j=1}^n a_n(j)$. Denoting $T_{n1}(\tau_0)$ such a form of $T_n(\tau_0)$, the mutual relation is given by $|T_n(\tau_0) - T_{n1}(\tau_0)| = o_{\mathrm{P}}((\log\log n)^{-1/2})$, as $n \to \infty$. Since $\bar{a}_n = 0$ for the scores $a_n(j)$ defined in (2.3) we have

$$T_{n1} = T_{n1}(\tau_0) = \max_{1 \le k < n} \sqrt{\frac{n}{k(n-k)}}\,\frac{1}{\sigma_n}\left|\sum_{j=1}^k a_n(j)\right|,$$

where $\sigma_n^2 = \frac{1}{n-1}\sum_{j=1}^n (a_n(j))^2$.

Theorem 3.1. *Suppose $X_1^0, \dots, X_n^0$, $C_1, \dots, C_n$ are independent random variables. Let $X_1^0, \dots, X_n^0$ have arbitrary continuous distribution function F. Let $C_1, \dots, C_n$ have the continuous distribution function $G = 1-(1-F)^\lambda$ for some $\lambda > 0$. Let* (2.4) *be satisfied and let*

$$\int_0^{\tau_0} w(t)(1-G(t))\,\mathrm{d}F(t) > 0, \quad i = 1, 2. \tag{3.1}$$

If $n \to \infty$, then for all $y \in \mathbb{R}$ we have

$$\mathrm{P}\Big(d_1(\log n)\ T_{n1} \le y + d_2(\log n)\Big) \to \exp\big\{-2\mathrm{e}^{-y}\big\}, \tag{3.2}$$

where d_1 and d_2 are defined in (2.7).

Proof. Realize that the random variables $\sum_{j=1}^k a_n(j)$, $k = 1, \dots, n$, have the same distribution as $\sum_{j=1}^k a_n(R_j)$, where $\mathcal{R} = (R_1, \dots, R_n)$ is a random permutation of $(1, \dots, n)$ and $\sum_{j=1}^k a_n(R_j)$, $k = 1, \dots, n$, given $(X_1, \triangle_1), \dots, (X_n, \triangle_n)$, can be viewed as simple linear rank statistics. Their expectations are zero and variances are $\frac{k(n-k)}{n}\sigma_n^2$. By Lemma 2.1 (special case $\eta = 1$) in [7] and (3.1), the scores defined in (2.3) fulfil for convergence in probability

$$\liminf_{n\to\infty} \frac{1}{n}\sum_{j=1}^n (a_n(j) - \bar{a}_n)^2 \ge D_1, \qquad \limsup_{n\to\infty} \frac{1}{n}\sum_{j=1}^n |a_n(j) - \bar{a}_n|^{2+u} \le D_2 \tag{3.3}$$

for some positive D_1, D_2 and $u = 2$. Thus Theorem 2 in [6] can be applied and we get

$$\mathrm{P}\left(d_1(\log n)\, T_{n1}(\mathcal{R}) \le y + d_2(\log n) | (X_1, \triangle_1), \dots, (X_n, \triangle_n)\right) \xrightarrow{\mathrm{P}} \exp\left\{-2\mathrm{e}^{-y}\right\},$$

Since the limit distribution does not depend on the condition $(X_1, \triangle_1), \dots, (X_n, \triangle_n)$ we can conclude that (3.2) holds true. □

Next we describe the permutation test related to the statistic T_{n1} and we will study its limit performance. More information about permutation tests for changes in location can be found in [2], e.g.

The permutation distribution of T_{n1} can be described as the conditional distribution given $(X_1, \triangle_1), \dots, (X_n, \triangle_n)$ of

$$T_{n1}(\mathcal{R}) = \max_{1 \le k < n} \sqrt{\frac{n}{k(n-k)}} \frac{1}{\sigma_n} \left| \sum_{j=1}^k a_n(R_j) \right|.$$

This permutation distribution $F_n(\cdot, (\boldsymbol{X}, \boldsymbol{\Delta})) = F_n(\cdot, (X_1, \triangle_1), \dots, (X_n, \triangle_n))$ can be expressed as

$$F_n(x, (\boldsymbol{X}, \boldsymbol{\Delta})) = \frac{1}{n!}\,\#\{\boldsymbol{r} \in \mathcal{R}_n; T_{n1}(\boldsymbol{r}) \le x\}, \quad x \in \mathbb{R},$$

where $\mathcal{R}_n$ is the set of all permutations of $(1, \dots, n)$ and $\#A$ denotes the cardinality of a set A. Denoting by $x_n(\alpha, (\boldsymbol{X}, \boldsymbol{\Delta}))$ the corresponding $(1-\alpha)$-quantile, the critical region of the permutation test based on T_{n1} with the level α has the form

$$T_{n1} \ge x_n(\alpha, (\boldsymbol{X}, \boldsymbol{\Delta})).$$

Next we derive the limit distribution of the permutation distribution $F_n(\cdot, (\boldsymbol{X}, \boldsymbol{\Delta}))$.

Theorem 3.2. *Suppose the KG model of random censorship. Let* $X_1^0, \dots, X_{[n\gamma]}^0$ *have continuous distribution function* F_1 *and* $X_{[n\gamma]+1}^0, \dots, X_n^0$ *have continuous distribution function* F_2 *for some* $\gamma \in (0,1]$. *Let* (2.4) *be satisfied and let*

$$\int_0^{\tau_0} w(t)(1-F_i(t))^\lambda \, \mathrm{d}F_i(t) > 0, \quad i=1,2. \tag{3.4}$$

If $n \to \infty$, *then for all* $y \in \mathbb{R}$ *we have*

$$\mathrm{P}\Big(d_1(\log n)\, T_{n1}(\mathcal{R}) \le y + d_2(\log n) | (X_1, \triangle_1), \dots, (X_n, \triangle_n)\Big) \xrightarrow{\mathrm{P}} \exp\big\{-2\mathrm{e}^{-y}\big\},$$

where d_1 *and* d_2 *are defined in* (2.7).

Proof. Denote $m = [n\gamma]$. Realize that $T_{n1}(\mathcal{R})$ given $(X_1, \triangle_1), \dots, (X_n, \triangle_n)$ can be viewed as a function of a simple linear rank statistic. Verify (3.3):
By a slight modification of the proof of Lemma 2.1 in [7] we obtain, as $n \to \infty$,

$$\sup_{0 \le x \le \tau_0} \Bigg| \int_0^x \frac{w_n(t)}{Y(t)} \, \mathrm{d}N(t) - \int_0^x \frac{w(t)\, \gamma\, (1-F_1(t))^\lambda}{\gamma(1-F_1(t))^{\lambda+1} + (1-\gamma)(1-F_2(t))^{\lambda+1}} \, \mathrm{d}F_1(t)$$
$$- \int_0^x \frac{w(t)\, (1-\gamma)\, (1-F_2(t))^\lambda}{\gamma(1-F_1(t))^{\lambda+1} + (1-\gamma)(1-F_2(t))^{\lambda+1}} \, \mathrm{d}F_2(t) \Bigg| = o_{\mathrm{P}}(1).$$

Moreover, by (2.4), $\sup_{1 \le j \le n} | \int_0^{\tau_0} w_n(t) \, \mathrm{d}N_j(t) - \int_0^{\tau_0} w(t) \, \mathrm{d}N_j(t) | = o_{\mathrm{P}}(1)$, as $n \to \infty$. Denoting

$$a^*(j) = \int_0^{\tau_0} w(t) \, \mathrm{d}N_j(t) - \int_0^{\tau_0} \frac{w(t)\, Y_j(t)\, \gamma\, (1-F_1(t))^\lambda}{\gamma(1-F_1(t))^{\lambda+1} + (1-\gamma)(1-F_2(t))^{\lambda+1}} \, \mathrm{d}F_1(t)$$
$$- \int_0^{\tau_0} \frac{w(t)\, Y_j(t)\, (1-\gamma)\, (1-F_2(t))^\lambda}{\gamma(1-F_1(t))^{\lambda+1} + (1-\gamma)(1-F_2(t))^{\lambda+1}} \, \mathrm{d}F_2(t)$$

we observe that, as $n \to \infty$, $\max_{1 \le j \le n} |a_n(j) - a^*(j)| = o_{\mathrm{P}}(1)$ and hence

$$\frac{1}{n} \sum_{i=1}^n (a_n(j) - a^*(j))^v = o_{\mathrm{P}}(1), \quad v = 1, 2, 3, \dots$$

The random variables $a^*(1), \dots, a^*(n)$ are bounded,

$$|a^*(j)| \le \max_{0 \le t \le \tau_0} |w(t)| \left(1 + \frac{1}{\gamma(1-F_1(\tau_0))^{\lambda+1} + (1-\gamma)(1-F_2(\tau_0))^{\lambda+1}}\right),$$

and $a^*(1), \dots, a^*(m)$ are i.i.d., $a^*(m+1), \dots, a^*(n)$ are i.i.d. Direct calculations give

$$\gamma\, \mathrm{E}\, a^*(1) + (1-\gamma)\, \mathrm{E}\, a^*(n) = 0,$$
$$\gamma\, \mathrm{E}(a^*(1))^2 + (1-\gamma)\, \mathrm{E}(a^*(n))^2 = \gamma \int_0^{\tau_0} w^2(t)(1-F_1(t))^\lambda \, \mathrm{d}F_1(t)$$
$$+ (1-\gamma) \int_0^{\tau_0} w^2(t)(1-F_2(t))^\lambda \, \mathrm{d}F_2(t) > 0.$$

The verification of (3.3) is then finished by application of the weak law of large numbers, so Theorem 2 in [6] can be applied. □

Remark. Notice that the assumptions of Theorem 3.2 cover both the null hypothesis and the alternative. Moreover, the limit permutation distribution is the same in both cases and does not depend on $(X_1, \triangle_1), \dots, (X_n, \triangle_n)$. This means that the critical value for the permutation test provides an approximation for the critical value to the test based on T_{n1}.

Under the hypothesis H_0 the distributions of T_{n1} and $T_{n1}(\mathcal{R})$ coincide and the permutation distribution provides the exact critical values for our testing problem.

4. Simulations

To compare the behaviour of the above described procedures for the testing a change in distribution of survival times, i.e., asymptotic and empirical critical values for T_n and T_{n1}, we have performed some simulations in which we had generated data from the location model, where we choose F as *exponential* $\mathrm{E}(\beta)$ and *lognormal* $\mathrm{L}(\beta)$ distribution

$$F(x) = 1 - \exp(-\beta x), \qquad \text{or} \qquad F(x) = \Phi(\log(\beta x)), \quad x > 0,$$

with parameter $\beta > 0$ and Φ standing for the standard normal distribution function.

We assume $\tau_0 = \infty$. In this case the test depends on X_j and Δ_j only:

$$W_k(\infty) = \frac{1}{\sqrt{n}} \sum_{j=1}^{k} a_n(j), \qquad a_n(j) = w_n(X_j)\,\Delta_j - \sum_{l=1}^{n} w_n(X_l)\,\Delta_l\,\frac{Y_j(X_l)}{Y(X_l)};$$

$$V_k(\infty) = \frac{1}{n} \sum_{l=1}^{n} w_n^2(X_l)\,\Delta_l\,\frac{\sum_{j=1}^{k} Y_j(X_l)\,\sum_{j=k+1}^{n} Y_j(X_l)}{Y^2(X_l)} + v_k;$$

$$w_n(X_l) = \left(\prod_{i:X_i<X_l} \left(1 - \frac{\Delta_i}{Y(X_i)}\right)\right)^{\rho} \left(\frac{Y(X_l)}{n}\right)^{\kappa}.$$

We use three types of weights for

(1) *log-rank*-type test (LR): $\rho = 0$, $\kappa = 0$;
(2) *Gehan–Wilcoxon*-type test (GW): $\rho = 0$, $\kappa = 1$;
(3) *Prentice–Wilcoxon*-type test (PW): $\rho = 1$, $\kappa = 0$.

The asymptotic critical values according to (2.6) and (3.2) for the chosen sample size $n = 120$ are summarized in Table 1.

n	10%	5%	2.5%	1%
120	3.236	3.643	4.042	4.564

Tab. 1 *Asymptotic critical values for T_n and T_{n1}.*

4.1. **Critical values for the statistic T_n.** Suppose the classical model of random censorship. Let us proceed with $n = 120$ as follows:

(1) $X_1^0, \dots, X_n^0$ are simulated from the chosen distribution F;
(2) $C_1, \dots, C_n$ are simulated using the chosen combination of parameters $C_i = \delta_n^C\, I(i > [n\eta]) + \epsilon_i$ for $i = 1, \dots, n$ (we use $\eta = 0.5$, $\delta_n^C = 0, 2$, $\epsilon_i \sim \mathrm{Unif}(0, b)$, $b = 1$ for $F = \mathrm{E}(1)$ and $b = 4$ for $F = \mathrm{L}(1)$);
(3) pairs $(X_1, \Delta_1), \dots, (X_n, \Delta_n)$ are computed;
(4) T_n is calculated and its value stored;
(5) the steps (1)-(4) are repeated 10^4 times;
(6) empirical quantiles related to the empirical distribution function of T_n are computed and used as an estimator of the actual quantiles.

In Table 2 the results of the simulation are summarized and in Table 3 the results of the simulation for the particular situation where survival variables $X_1^0, \dots, X_n^0$ are not censored can be found.

δ^C_{120}	$c_{120}(\alpha)$	exponential 10%	5%	2.5%	1%	lognormal 10%	5%	2.5%	1%
0	LR	3.460	4.142	4.733	6.153	3.518	4.173	5.003	6.429
0	GW	3.315	4.009	4.672	5.700	3.354	3.960	4.696	6.069
0	PW	3.411	4.074	4.752	6.049	3.415	4.070	4.895	6.022
2	LR	3.464	4.091	4.705	5.900	3.477	4.071	4.781	6.128
2	GW	3.329	3.939	4.678	5.999	3.378	4.036	4.834	6.015
2	PW	3.358	4.017	4.777	5.805	3.394	4.010	4.700	6.058

Tab. 2 *Empirical critical values for T_n.*

$c_{120}(\alpha)$	exponential 10%	5%	2.5%	1%	lognormal 10%	5%	2.5%	1%
LR	3.454	4.048	4.846	5.982	3.453	4.093	4.821	5.961
GW	3.298	3.921	4.637	5.602	3.374	3.999	4.726	5.856
PW	3.348	3.970	4.715	5.617	3.349	3.983	4.721	5.862

Tab. 3 *Empirical critical values for T_n - no censoring, no change.*

The empirical critical values are almost not influenced by the change in location of the distribution of censoring variables C_i's, the choice of the weights and the underlying distribution. Comparing the results in Table 2 and Table 3, we can see that the empirical critical values in case of censored survival variables X_i^0's are similar to their counterparts in case of uncensored X_i^0's. Surprisingly, the simulated critical values are substantially larger than the corresponding asymptotic ones, which is probably influenced by large variability of V_k defined in (2.2) and it needs another extended investigation.

4.2. **Critical values for the statistic T_{n1}.** Suppose the KG model of random censorship. Let us proceed with $n = 120$ as follows:

(1) $X_1^0, \ldots, X_n^0$ are simulated using the chosen combination of parameters $X_i^0 = \delta_n I(i > [n\gamma]) + \varepsilon_i$ for $i = 1, \ldots, n$ (we use $\gamma = 0.5$, $\delta_n = 0, 2$, $\varepsilon_i \sim F$, $F = \mathrm{E}(1)$ or $\mathrm{L}(1)$);
(2) $C_1, \ldots, C_n$ fulfilling the KG model are simulated (we use $\lambda = 0, 0.5, 1$);
(3) pairs $(X_1, \Delta_1), \ldots, (X_n, \Delta_n)$ are computed;
(4) a random permutation $\boldsymbol{r} = (r_1, \ldots, r_n)$ of $(1, \ldots, n)$ is generated;
(5) $T_{n1}(\mathcal{R})$ with $\mathcal{R} = \boldsymbol{r}$ is calculated and its value stored;
(6) the steps (4)-(5) are repeated 10^4 times;
(7) empirical quantiles related to the empirical distribution function of $T_{n1}(\mathcal{R})$ are computed and used as an estimator of the actual quantiles.

The empirical critical values for $T_{n1}(\mathcal{R})$ obtained through the permutation principle do reasonable approximation of the critical values for T_{n1}. In Table 4 the results of the simulation are presented.

The empirical critical values are almost not influenced by the change both in location model and the underlying distribution, and they are substantially smaller than the corresponding asymptotic ones. In case of no censoring the empirical critical values for the log-rank-type test are larger than for other two tests (and very similar to the asymptotic critical values), but in other cases we can see similar results for the considered weights. Comparing the obtained critical values with the results in [2], we see similar patterns. The difference between the empirical critical values for T_n and T_{n1} is caused by the choice of standardization of the statistic $W_k(\infty)$.

			exponential				lognormal			
δ_{120}	λ	$c_{120}(\alpha)$	10%	5%	2.5%	1%	10%	5%	2.5%	1%
0	0	LR	3.089	3.447	4.121	4.470	3.045	3.447	4.070	4.470
0	0	GW	2.752	2.994	3.216	3.462	2.748	2.987	3.196	3.451
0	0	PW	2.744	2.976	3.197	3.441	2.735	2.974	3.208	3.460
0	0.5	LR	2.800	3.082	3.302	3.617	2.861	3.222	3.532	3.718
0	0.5	GW	2.756	2.993	3.190	3.456	2.733	2.985	3.179	3.446
0	0.5	PW	2.757	2.986	3.193	3.450	2.759	2.982	3.200	3.477
0	1	LR	2.905	3.363	3.535	3.641	2.831	3.184	3.410	3.588
0	1	GW	2.835	3.088	3.321	3.577	2.852	3.119	3.386	3.630
0	1	PW	2.761	3.005	3.207	3.511	2.804	3.042	3.250	3.513
2	0	LR	3.033	3.447	3.963	4.470	3.042	3.447	3.984	4.470
2	0	GW	2.730	2.968	3.183	3.419	2.746	2.971	3.188	3.435
2	0	PW	2.735	2.963	3.170	3.428	2.737	2.982	3.198	3.477
2	0.5	LR	2.921	3.068	3.311	3.606	2.956	3.409	3.555	3.788
2	0.5	GW	2.784	3.007	3.233	3.545	2.745	2.977	3.210	3.458
2	0.5	PW	2.747	2.955	3.165	3.433	2.757	2.999	3.219	3.500
2	1	LR	2.977	3.152	3.350	3.658	2.772	3.038	3.257	3.528
2	1	GW	2.833	3.114	3.359	3.622	2.829	3.071	3.285	3.575
2	1	PW	2.775	3.010	3.246	3.537	2.756	3.003	3.196	3.450

Tab. 4 *Empirical critical values for $T_{n1}(\mathcal{R})$.*

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