SOME MODIFICATIONS OF RECURSIVE TIME SERIES METHODS

Tomáš Cipra, Charles University of Prague

Recursive procedures used for smoothing, estimating and predicting time series are very popular in the modern time series analysis due to their advantageous properties. The contribution shows some possible modifications of such procedures that include (1) robust recursive methods, (2) recursive methods in L_1-norm, (3) recursive methods with missing observations, (4) asymmetric recursive methods. Examples of the modified recursive procedures concern Kalman filter, exponential smoothing and autoregressive processes.

Keywords: recursive methods, time series, Kalman filter, exponential smoothing, AR processes, robust methods, L_1-norm, missing observations, asymmetry

AMS classification: 62M10, 62M20, 60G35

1. INTRODUCTION

Recursive methods in time series consist in adaptation of a previous estimate by means of a correction term which depends both on the previous estimate and on a new information (a new observation). They are used successfully for estimation, smoothing and prediction in time series analysis due to their advantageous properties (flexibility, numerical efficiency, memory saving and others).

Majority of recursive time series methods can be interpreted as special cases of the Kalman filter for the dynamic linear model (or the state-space representation) of the form

$$x_t = F_t x_{t-1} + w_t\,, \tag{1.1}$$

$$y_t = H_t x_t + v_t\,, \tag{1.2}$$

where

$$\mathrm{E}\, w_t = 0,\ \mathrm{E}\, v_t = 0,\ \mathrm{E}(w_s w_t') = \delta_{st} Q_t,\ \mathrm{E}(v_s v_t') = \delta_{st} R_t,\ \mathrm{E}(w_s v_t') = 0 \tag{1.3}$$

and some initial conditions are fulfilled. The state equation (1.1) describes behavior of an n-dimensional state vector x_t in time while the observation equation (1.2) describes relation of the unobservable state x_t to an m-dimensional observation vector y_t.

The Kalman filter gives recursive formulas for construction of the linear minimum variance estimator $\hat{x}_t^t$ of x_t and its error covariance matrix $P_t^t = \mathrm{E}(x_t - \hat{x}_t^t)(x_t - \hat{x}_t^t)'$ at time t using all previous information $\{y_0, y_1, \dots, y_t\}$:

$$\hat{x}_t^t = \hat{x}_t^{t-1} + P_t^{t-1} H_t' (H_t P_t^{t-1} H_t' + R_t)^{-1} (y_t - H_t \hat{x}_t^{t-1})\,, \tag{1.4}$$

$$P_t^t = P_t^{t-1} - P_t^{t-1} H_t' (H_t P_t^{t-1} H_t' + R_t)^{-1} H_t P_t^{t-1}\,, \tag{1.5}$$

where

$$\hat{x}_t^{t-1} = F_t \hat{x}_{t-1}^{t-1}, \tag{1.6}$$

$$P_t^{t-1} = F_t P_{t-1}^{t-1} F_t' + Q_t \tag{1.7}$$

are predictive values for time t at time $t-1$. In the standard normal case with

$$w_t \sim \mathrm{N}(0, Q_t), \quad v_t \sim \mathrm{N}(0, R_t) \tag{1.8}$$

the Kalman filter provides the minimum variance estimators: e.g. it holds $\hat{x}_t^t = \mathrm{E}(x_t | y_0, y_1, \ldots, y_t)$ in this case.

It can be easily shown that the state estimator $\hat{x}_t^t$ can be obtained by solving the optimization problem

$$\hat{x}_t^t = \mathrm{argmin}\{(\hat{x}_t^{t-1} - x_t)'(P_t^{t-1})^{-1}(\hat{x}_t^{t-1} - x_t) + (y_t - H_t x_t)' R_t^{-1}(y_t - H_t x_t)\} \tag{1.9}$$

over $x_t \in \mathbb{R}^n$, or equivalently

$$\hat{x}_t^t = \mathrm{argmin}\{\sum_{i=1}^{n}(p_{it} - a_{it}x_t)^2 + \sum_{j=1}^{m}(s_{jt} - b_{jt}x_t)^2\}, \tag{1.10}$$

where $p_t = (P_t^{t-1})^{-\frac{1}{2}}\hat{x}_t^{t-1}$, $s_t = R_t^{-\frac{1}{2}} y_t$, $a_t = (P_t^{t-1})^{-\frac{1}{2}}$, $b_t = R_t^{-\frac{1}{2}} H_t$ (e.g., p_{it} is the i-th component of the vector p_t and a_{it} is the i-th row of the matrix a_t).

From the point of view of practical applications some modifications of the classical recursive methods may be important. This contribution shows such modifications including robust recursive methods (see Section 2), recursive methods in L_1-norm (see Section 3), recursive methods with missing observations (see Section 4) and asymmetric recursive methods (see Section 5).

2. ROBUST RECURSIVE METHODS

The assumption (1.8) may not be fulfilled in practice where various forms of contaminated data can be expected. Therefore robustification of the Kalman filter is important for practical applications. Among numerous methods suggested in the literature the robust modification of the Kalman filter based on the methodology of the M-estimators seems to give results acceptable from the numerical point of view (see Cipra, Romera (1991)). In this case one replaces the least squares problem (1.10) by

$$\hat{x}_t^t = \mathrm{argmin}\{\sum_{i=1}^{n} \varrho_{1i}(p_{it} - a_{it}x_t) + \sum_{j=1}^{m} \varrho_{2j}(s_{jt} - b_{jt}x_t)\}, \tag{2.1}$$

where ϱ_{1i} and ϱ_{2j} are suitable loss functions with derivatives ψ_{1i} and ψ_{2j} (the so--called psi-functions) used in robust statistics.

In order to obtain explicit results we confine ourselves to the special case of contaminated scalar observations, where $m = 1$ (i.e. we have a row n-dimensional vector h_t instead of the matrix H_t in (1.2)) and the assumptions (1.8) is replaced by

$$w_t \in \mathrm{N}(0, Q_t), \quad v_t \sim \varepsilon\text{-contaminated } \mathrm{N}(0, r_t). \tag{2.2}$$

The ε-contamination in (2.2) means that the normal distribution with an acceptable variance r_t is contaminated by a small fraction ε (e.g. $\varepsilon = 0.05$) of a symmetric distribution with heavy tails that is the source of outliers in scalar observations $\{y_t\}$. For such data with ε-contaminated distribution N(0,1) (the unit variance can be achieved by means of standardization) the Huber's psi-function ψ_H of the form

$$\psi_H(z) = \begin{cases} z & \text{for } |z| \le c \\ c \,\mathrm{sgn}(z) & \text{for } |z| > c \end{cases} \tag{2.3}$$

gives robust estimates of location that are optimal in the min-max sense having the minimal variance over the least favorable contaminating distributions. The recommended choice of c in (2.3) is $c = u_{1-\varepsilon}$, where u_α is the α-quantile of N(0,1) (e.g. $c = 1.645$ for the 5% contamination of $\{y_t\}$).

If we put $m = 1$, $\psi_{1i}(z) = \psi_{LS}(z) = z$, $\psi_{21} = \psi_H(z)$ in (2.1) then we obtain the following robust recursive formula

$$\hat{x}_t^t = \hat{x}_t^{t-1} + P_t^{t-1} h_t' r_t^{-\frac{1}{2}} \psi_H \left(\frac{r_t^{\frac{1}{2}} (y_t - h_t \hat{x}_t^{t-1})}{h_t P_t^{t-1} h_t' + r_t} \right). \tag{2.4}$$

The exact recursive formula for P_t^t is too complicated so that one recommends to use approximatively the formula of the classical Kalman filter

$$P_t^t = P_t^{t-1} - \frac{P_t^{t-1} h_t' h_t P_t^{t-1}}{h_t P_t^{t-1} h_t' + r_t}. \tag{2.5}$$

Some special cases may be useful in practice:

(1) Recursive estimation in AR(1) process with innovation outliers: In an AR(1) process

$$y_t = \varphi y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \varepsilon\text{-contaminated } iid\, \mathrm{N}(0, \sigma^2) \tag{2.6}$$

with the corresponding state-space representation

$$x_t = x_{t-1}, \tag{2.7}$$

$$y_t = y_{t-1} x_t + \varepsilon_t \tag{2.8}$$

(i.e. $F_t = 1$, $H_t = h_t = y_{t-1}$) one obtains according to (1.6), (1.7), (2.4) and (2.5) the recursive estimation formulae

$$\hat{x}_t^t = \hat{x}_{t-1}^{t-1} + P_{t-1}^{t-1} y_{t-1} \sigma^{-1} \psi_H \left(\frac{\sigma (y_t - y_{t-1} \hat{x}_{t-1}^{t-1})}{P_{t-1}^{t-1} y_{t-1}^2 + \sigma^2} \right), \tag{2.9}$$

$$P_t^t = \frac{P_{t-1}^{t-1} \sigma^2}{P_{t-1}^{t-1} y_{t-1}^2 + \sigma^2}. \tag{2.10}$$

It can be proved under very weak assumptions that

$$\hat{x}_t^t \to \varphi \text{ a. s.} \tag{2.11}$$

(see Cipra, Romera (1991)). The work Cipra, Rubio, Canal (1993) describes a recursive procedure for autoregressive processes with additive outliers.

(2) Robust Holt method: The Holt method is a practical smoothing and predicting method recommended for time series with a locally linear trend. The corresponding dynamic linear model has the form

$$L_t = L_{t-1} + T_{t-1} + \partial L_t, \quad \partial L_t \sim iid\,\mathrm{N}(0, \sigma_1^2), \tag{2.12}$$

$$T_t = T_{t-1} + \partial T_t, \quad \partial T_t \sim iid\,\mathrm{N}(0, \sigma_2^2), \tag{2.13}$$

$$y_t = L_t + \varepsilon_t, \quad \varepsilon_t \sim \varepsilon\text{-contaminated } iid\,\mathrm{N}(0, \sigma^2), \tag{2.14}$$

where L_t and T_t denote the level and the trend at time t, respectively. The residuals $\{\partial L_t\}$, $\{\partial T_t\}$ and $\{\varepsilon_t\}$ are supposed to be mutually independent. If applying the previous scheme one obtains the robust Holt method of the form

$$\hat{L}_t^t = \hat{L}_{t-1}^{t-1} + \hat{T}_{t-1}^{t-1} + \frac{\alpha}{(1-\alpha)^{\frac{1}{2}}} s_t \psi_H\left((1-\alpha)^{\frac{1}{2}} \frac{e_t}{s_t}\right), \tag{2.15}$$

$$\hat{T}_t^t = \hat{T}_{t-1}^{t-1} + \frac{\alpha\gamma}{(1-\alpha)^{\frac{1}{2}}} s_t \psi_H\left((1-\alpha)^{\frac{1}{2}} \frac{e_t}{s_t}\right), \tag{2.16}$$

$$\hat{y}_{t+k}^t = \hat{L}_t^t + k\hat{T}_t^t \text{ for } k \geq 0, \tag{2.17}$$

where $\alpha, \gamma \in (0,1)$ are smoothing constants from the classical Holt method, $e_t = y_t - \hat{y}_t^{t-1}$ is the one-step-ahead prediction error and s_t is a suitable estimate of standard deviation $\sigma(e_t^{t-1})$, e.g.

$$s_t = 1.25\varkappa|e_t| + (1-\varkappa)s_{t-1}, \tag{2.18}$$

where $\varkappa \in (0,1)$ is a constant chosen near to zero and 1.25 approximates $(\pi/2)^{\frac{1}{2}}$ (for details see Cipra, Rubio, Canal (1992)).

(3) Robust Holt-Winters method: This method generalizes the Holt method for seasonal data. E.g. in the additive case, the corresponding dynamic linear model (called basic structural model in this context) has the form

$$L_t = L_{t-1} + T_{t-1} + \partial L_t, \quad \partial L_t \sim iid\,\mathrm{N}(0, \sigma_1^2), \tag{2.19}$$

$$T_t = T_{t-1} + \partial T_t, \quad \partial T_t \sim iid\,\mathrm{N}(0, \sigma_2^2), \tag{2.20}$$

$$I_t = -\sum_{i=1}^{p-1} I_{t-i} + \partial I_t, \quad \partial I_t \sim iid\,\mathrm{N}(0, \sigma_3^2), \tag{2.21}$$

$$y_t = L_t + I_t + \varepsilon_t, \quad \varepsilon_t \sim \varepsilon\text{-contaminated } iid\,\mathrm{N}(0, \sigma^2), \tag{2.22}$$

where p is the length of season, I_t is the seasonal index at time t and the residuals $\{\partial L_t\}$, $\{\partial T_t\}$, $\{\partial I_t\}$ and $\{\varepsilon_t\}$ are supposed to be mutually independent. The corresponding recursive formulae of the robust Holt-Winters method including a real data example are given in Cipra, Rubio, Canal (1992).

3. RECURSIVE METHODS IN L_1-NORM

If choosing the loss functions ϱ_{1i} and ϱ_{2j} in (2.1) as absolute values one obtains the Kalman filter in L_1-norm (see Cipra, Romera (1992)). Here we confine ourselves only to two special cases comparing the results with the ones provided by the classical least squares approach (i.e. by the L_2-norm).

(1) Steady model in L_1-norm: The steady model applicable e.g. in finance has the form

$$x_t = x_{t-1} + w_t\,, \quad \operatorname{var}(w_t) = q_t\,, \tag{3.1}$$

$$y_t = x_t + v_t\,, \quad \operatorname{var}(v_t) = r_t \tag{3.2}$$

under assumptions (1.3). The corresponding formulae provided by the Kalman filter in L_1-norm are

$$\tilde{x}_t^t = \tilde{A}_t y_t + (1-\tilde{A}_t)\tilde{x}_{t-1}^{t-1}\,, \tag{3.3}$$

$$\tilde{P}_t^t = \tilde{A}_t r_t + (1-\tilde{A}_t)(\tilde{P}_{t-1}^{t-1} + q_t)\,, \tag{3.4}$$

where

$$\tilde{A}_t = \begin{cases} 1 & \text{for } r_t < \tilde{P}_{t-1}^{t-1} + q_t \\ 0 & \text{for } r_t \geq \tilde{P}_{t-1}^{t-1} + q_t. \end{cases} \tag{3.5}$$

One can see that the state estimator $\tilde{x}_t^t$ for the steady model in L_1-norm is restricted to two values only, namely $\tilde{x}_{t-1}^{t-1}$ or y_t. It can be compared with the following results for the classical steady model in L_2-norm

$$\hat{x}_t^t = A_t y_t + (1-A_t)\hat{x}_{t-1}^{t-1}\,, \tag{3.6}$$

$$P_t^t = A_t r_t + (1-A_t)(P_{t-1}^{t-1} + q_t)\,, \tag{3.7}$$

where

$$A_t = \frac{P_{t-1}^{t-1} + q_t}{P_{t-1}^{t-1} + q_t + r_t}. \tag{3.8}$$

It is obviously

$$A_t \in \begin{cases} (\frac{1}{2}, 1) & \text{for } r_t < P_{t-1}^{t-1} + q_t \\ (0, \frac{1}{2}) & \text{for } r_t \geq P_{t-1}^{t-1} + q_t. \end{cases} \tag{3.9}$$

(2) Recursive estimation in AR(1) process in L_1-norm: The Kalman filter in the corresponding dynamic linear model (see also (2.7), (2.8))

$$x_t = x_{t-1} \tag{3.10}$$

$$y_t = y_{t-1} x_t + \varepsilon_t\,, \quad \operatorname{var} \varepsilon_t = \sigma^2 \tag{3.11}$$

gives in L_1-norm the following recursive formulae

$$\tilde{x}_t^t = \begin{cases} \tilde{B}_t \frac{y_t}{y_{t-1}} + (1-\tilde{B}_t)\tilde{x}_{t-1}^{t-1} & \text{for } y_{t-1} \neq 0 \\ \tilde{x}_{t-1}^{t-1} & \text{for } y_{t-1} = 0\,, \end{cases} \tag{3.12}$$

$$\tilde{P}_t^t = \begin{cases} \tilde{B}_t \frac{\sigma^2}{y_{t-1}^2} + (1-\tilde{B}_t)\tilde{P}_{t-1}^{t-1} & \text{for } y_{t-1} \neq 0 \\ \tilde{P}_{t-1}^{t-1} & \text{for } y_{t-1} = 0\,, \end{cases} \tag{3.13}$$

where

$$\tilde{B}_t = \begin{cases} 1 & \text{for } \sigma^2 < y_{t-1}^2 \tilde{P}_{t-1}^{t-1} \\ 0 & \text{for } \sigma^2 \geq y_{t-1}^2 \tilde{P}_{t-1}^{t-1}. \end{cases} \tag{3.14}$$

The parameter estimator $\tilde{x}_t^t$ for the AR(1) process in L_1-norm is again restricted to two values only, namely y_t/y_{t-1} or $\tilde{x}_{t-1}^{t-1}$. It can be compared with the following results in L_2-norm

$$\hat{x}_t^t = \begin{cases} B_t \frac{y_t}{y_{t-1}} + (1-B_t)\hat{x}_{t-1}^{t-1} & \text{for } y_{t-1} \neq 0 \\ \hat{x}_{t-1}^{t-1} & \text{for } y_{t-1} = 0\,, \end{cases} \tag{3.15}$$

$$P_t^t = \begin{cases} B_t \frac{\sigma^2}{y_{t-1}^2} + (1-B_t)P_{t-1}^{t-1} & \text{for } y_{t-1} \neq 0 \\ P_{t-1}^{t-1} & \text{for } y_{t-1} = 0\,, \end{cases} \tag{3.16}$$

where

$$B_t = \frac{y_{t-1}^2 P_{t-1}^{t-1}}{y_{t-1}^2 P_{t-1}^{t-1} + \sigma^2}\,. \tag{3.17}$$

It is obviously

$$B_t \in \begin{cases} (\frac{1}{2}, 1) & \text{for } \sigma^2 < y_{t-1}^2 P_{t-1}^{t-1} \\ (0, \frac{1}{2}) & \text{for } \sigma^2 \geq y_{t-1}^2 P_{t-1}^{t-1}\,. \end{cases} \tag{3.18}$$

4. RECURSIVE METHODS WITH MISSING OBSERVATIONS

As an example of recursive methods with missing observations we shall give the modification of the Holt-Winters method that enables smoothing and predicting seasonal data with missing observations (see Cipra, Trujillo, Rubio (1995)). We shall confine ourselves to the additive case only since the multiplicative case is analogical.

Let seasonal data $\{y_t\}$ with the length of season p be observable only at times $t_1 < t_2 < \cdots < t_n$. Let the symbols L_t, T_t and I_t denote the level, trend and seasonal index at time t, respectively (see also (2.19)–(2.22)). In order to estimate these values the following recursive formulae can be used modifying the classical Holt-Winters method:

$$\hat{L}_{t_n} = U_{t_n}(y_{t_n} - I_{t_n^*}) + (1 - U_{t_n})[\hat{L}_{t_{n-1}} + (t_n - t_{n-1})\hat{T}_{t_{n-1}}]\,, \tag{4.1}$$

$$\hat{T}_{t_n} = V_{t_n}\frac{\hat{L}_{t_n} - \hat{L}_{t_{n-1}}}{t_n - t_{n-1}} + (1 - V_{t_n})\hat{T}_{t_{n-1}}\,, \tag{4.2}$$

$$\hat{I}_{t_n} = W_{t_n}(y_{t_n} - \hat{L}_{t_n}) + (1 - W_{t_n})\hat{I}_{t_n^*} \tag{4.3}$$

with

$$(4.4) \qquad U_{t_n} = \frac{U_{t_{n-1}}}{a_{t_n} + U_{t_{n-1}}}, \quad a_{t_n} = (1-\alpha)^{t_n - t_{n-1}},$$

$$(4.5) \qquad V_{t_n} = \frac{V_{t_{n-1}}}{c_{t_n} + V_{t_{n-1}}}, \quad c_{t_n} = (1-\gamma)^{t_n - t_{n-1}},$$

$$(4.6) \qquad W_{t_n} = \frac{W_{t_{n-1}}}{d_{t_n} + W_{t_{n-1}}}, \quad d_{t_n} = (1-\delta)^{t_n - t_{n-1}},$$

where t_n^* denotes the largest value among t_{n-1}, t_{n-2}, ... such that the time t_n^* corresponds to the same seasonal period as the time t_n (e.g. for monthly observations, if t_n is a January value then t_n^* must be looked for among the past observable January values only). The constants $\alpha, \gamma, \delta \in (0,1)$ would be used as the smoothing constants by the classical Holt-Winters method in the case without missing observations. The smoothing constants U, V and W used in the previous modification with missing observations must be recalculated recursively according to (4.4)–(4.6).

The estimated level, trend and seasonal indices enables us to calculate the predicted values for m steps ahead

$$(4.7) \qquad \hat{y}_{t_n+m}^{t_n} = \hat{L}_{t_n} + m\hat{T}_{t_n} + \hat{I}_{(t_n+m)^*} \text{ for } m = 1, 2, \ldots,$$

the smoothed values

$$(4.8) \qquad \hat{y}_{t_n} = \hat{L}_{t_n} + \hat{I}_{t_n}$$

and the interpolated values

$$(4.9) \qquad \hat{y}_t = \hat{y}_t^{t_{n-1}} \text{ for } t = t_{n-1}+1, \ldots, t_n - 1,$$

where $(t_n+m)^*$ in (4.7) denotes the largest value among $t_n, t_{n-1}, \ldots$ such that the time $(t_n+m)^*$ corresponds to the same seasonal period as the time t_n+m. The details can be found in Cipra, Trujillo, Rubio (1995) including real data examples that provide acceptable results even in the cases with 50% missing observations. Some theoretical results concerning the exponential smoothing methods with missing observations are derived in Cipra (1989).

5. ASYMMETRIC RECURSIVE METHODS

A simple asymmetric model useful for practical applications is the dynamic linear model (1.1), (1.2) with scalar observations (i.e. $m = 1$) and

$$(5.1) \qquad v_t \sim \mathrm{N}(0; r_{1t}, r_{2t}).$$

The symbol $\mathrm{N}(0; \sigma_1^2, \sigma_2^2)$ denotes the split-normal distribution with the density

$$(5.2) \qquad f(x) = \begin{cases} \frac{2\sigma_2}{(\sigma_1+\sigma_2)\sigma_1}\varphi(\frac{x}{\sigma_1}) & \text{for } x < 0 \\ \frac{2\sigma_1}{(\sigma_1+\sigma_2)\sigma_2}\varphi(\frac{x}{\sigma_2}) & \text{for } x \geq 0, \end{cases}$$

where φ is the density of N(0,1) (see Cipra (1994)). If using this model one can replace the symmetric least squares estimation (1.9) by the asymmetric ones of the form

$$(5.3)\quad \hat{x}_t^t = \text{argmin}\{(\hat{x}_t^{t-1} - x_t)'(P_t^{t-1})^{-1}(\hat{x}_t^{t-1} - x_t) + r_{1t}^{-1}[(y_t - h_t x_t)^-]^2 + r_{2t}^{-1}[(y_t - h_t x_t)^+]^2\}.$$

It provides the recursive formula

$$(5.4)\quad \hat{x}_t^t = \hat{x}_t^{t-1} + \frac{P_t^{t-1}h_t'}{h_t P_t^{t-1} h_t' + r_{1t}}(y_t - h_t\hat{x}_t^{t-1})^- + \frac{P_t^{t-1}h_t'}{h_t P_t^{t-1} h_t' + r_{2t}}(y_t - h_t\hat{x}_t^{t-1})^+.$$

Let us consider some special cases:

(1) Asymmetric Holt method:

$$(5.5)\quad \hat{L}_t = \hat{L}_{t-1} + \hat{T}_{t-1} + \alpha_1 e_t^- + \alpha_2 e_t^+,$$

$$(5.6)\quad \hat{T}_t = \hat{T}_{t-1} + \alpha_1\gamma_1 e_t^- + \alpha_2\gamma_2 e_t^+,$$

$$(5.7)\quad \hat{y}_{t+k}^t = \hat{L}_t + k\hat{T}_t \text{ for } k \geq 0,$$

where $\alpha_1, \alpha_2, \gamma_1, \gamma_2 \in (0,1)$ are smoothing constants and $e_t = y_t - \hat{y}_t^{t-1}$ is the one--step-ahead prediction error. For $\alpha_1 = \alpha_2$ and $\gamma_1 = \gamma_2$ the method becomes the classical (symmetric) Holt method.

(2) Recursive estimation in AR(1) process with asymmetric innovations: In an AR(1) process

$$(5.8)\quad y_t = \varphi y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim iid\,\mathrm{N}(0;\sigma_1^2,\sigma_2^2)$$

with the state-space representation (2.7), (2.8) one obtains the recursive estimation formula

$$(5.9)\quad \hat{x}_t^t = \hat{x}_{t-1}^{t-1} + \frac{P_{t-1}^{t-1}y_{t-1}}{P_{t-1}^{t-1}y_{t-1}^2 + \sigma_1^2}(y_t - y_{t-1}\hat{x}_{t-1}^{t-1})^- + \frac{P_{t-1}^{t-1}y_{t-1}}{P_{t-1}^{t-1}y_{t-1}^2 + \sigma_2^2}(y_t - y_{t-1}\hat{x}_{t-1}^{t-1})^+.$$

If one trims the prediction error $e_t = y_t - y_{t-1}\hat{x}_{t-1}^{t-1}$ in a suitable way one can prove a similar consistency result as (2.11) (see Cipra (1994)).

References

Cipra T., *Some problems of exponential smoothing*, Aplikace matematiky **34**, 1989, 161–169.

Cipra T., *Robust exponential smoothing*, Journal of Forecasting **11**, 1992, 57–69.

Cipra T., *Asymmetric recursive methods for time series*, Appl. Math. 39, 1994, 203-214.

Cipra T., Romera R., *Robust Kalman filter and its application in time series analysis*, Kybernetika **27**, 1991, 481–494.

Cipra T., Romera R., *Recursive time series methods in L_1-norm*, In: L_1-Statistical Analysis and Related Methods (Y.Dodge, ed.), North Holland, Amsterdam 1992, 233–243.

Cipra T., Rubio A., Canal J.L., *Robustified smoothing and forecasting procedures*, Czechoslovak Journal of Operations Research **1**, 1992, 41–56.

Cipra T., Rubio A., Canal J.L., *Recursive estimation in autoregressive models with additive outliers*, Kybernetika **29**, 1993, 62–72.

Cipra T., Trujillo J., Rubio A., *Holt-Winters method with missing observations*, Management Science **41**, 1995, 174-178.

Anderson, B. D. O., Moore, J. B., *Optimal Filtering*, Prentice-Hall, Englewood Cliffs 1979.
Brockwell, P. J., Davis, R. A., *Introduction to Time Series and Forecasting*, Springer, New York 1996.
Campbell, K., *Recursive computation of M-estimates for the parameters of a finite autoregressive process*, Annals of Statistics **10**, 1982, 442–453.

The research was supported by the grant GA ČR 201/97/1176 „Časové řady a příbuzné modely“.

Author's address: Doc. RNDr. Tomáš Cipra, DrSc., Dept. of Statistics, Charles University, Sokolovská 83, 186 00 Praha 8, Czech Republic (cipra@karlin.mff.cuni.cz)

NĚKTERÉ MODIFIKACE REKURENTNÍCH METOD PRO ČASOVÉ ŘADY

V moderní analýze časových řad jsou velmi oblíbené pro své výhodné vlastnosti rekurentní metody vyrovnávání, odhadování a předpovídání. Příspěvek ukazuje některé možné modifikace těchto procedur: (1) robustní rekurentní metody, (2) rekurentní metody v L_1-normě, (3) rekurentní metody s chybějícími pozorováními, (4) asymetrické rekurentní metody. Příklady modifikovaných rekurentních metod se týkají Kalmanova filtru, exponenciálního vyrovnávání a autoregresních procesů.

НЕКОТОРЫЕ МОДИФИКАЦИИ РЕКУРЕНТНЫХ МЕТОДОВ ВО ВРЕМЕННЫХ РЯДАХ

В современном анализе временных рядов кажутся очень популярными для своих полезных свойств рекурентные методы сглаживания, оценивания и предикции. В работе предложены некоторые возможные модификации этих процедур: (1) робустные рекурентные методы, (2) рекурентные методы в L_1-норме, (3) рекурентные методы с отсутствующими набюдениями, (4) несимметричные рекурентные методы. Примеры модифицированных рекурентных методов касаются фильтра Калмана, экспоненциального сглаживания и процессов авторегрессии.