

SOFTWARE FOR STOCHASTIC APPROXIMATION

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SUMMARY

The full menu integrated programed system developed for IBM XT (AT) compatibles will be introduced. This program system offers several methods for solving problems of one dimensional stochastic approximation both for the root and the extreme estimates. It can be used also for the root and the extreme estimates of the function of location parameters or regression quantiles function, in particular. The estimation of parameters of the unknown distribution together with solution of the LD-50 problem is included as the special part of the program. The program offers the basic statistics and information for comparisons of different methods chosen by the user including graphical information. The theoretical background for all the methods can be found in the cited literature. More detailed description of the program is given in author's PhD. thesis.

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The following problems can be solved by the program. The problems those are solvable by the program can be divided into several groups. In the mathematical formulation of the problems we shall use the following notation:

(R, \mathcal{B}) ... one-dimensional Euclidean measurable space with Borel σ -algebra,
 E ... expectation functional on some probability space (Ω, \mathcal{A}, P) ,
 $X(t)$... the value of the estimator at time t ,
 $Y(x)$... the value of the measurement at the point x (this value can be generated by the random numbers generator).

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A/Problem of finding the root of the regression function

The problem can be formulated as solving the equation

$$Eg(x, \cdot) = \alpha, \quad /1/$$

where $g: (R \times \Omega, \mathcal{B} \otimes \mathcal{A}) \rightarrow (R, \mathcal{B})$ and α is some given real number which can be chosen arbitrarily in the program. The function g can also be defined by user.

Methods available for solving of problem /1/:

- i/ Robbins-Monro procedure (Robbins-Monro [1951], Blum [1954])
- ii/ Adaptive Robbins-Monro procedure (Fabian [1968])
- iii/ Procedure based on isotonic regression (Mukerjee [1981], Charamza [1984])
- iv/ Procedure based on quasiisotonic regression (Dupač [1987])
- v/ Procedure based on L1-quasiisotonic regression (Charamza [1990])
- vi/ Pflug procedure (Pflug [1988])

B/Problem of finding the root of the regression quantile function

The procedures of this paragraph solves iteratively the equation

$$Q(x, \gamma) = \alpha, \quad /2/$$

where $Q(x, \gamma)$ is defined as the γ -quantile of the distribution according to which the observations at the point x are obtained.

i/ Procedure based on isotonic quantile regression (Charamza [1990])

ii/ Procedure based on quasiisotonic quantile regression (Charamza [1990])

iii/ Derman procedure (Derman [1954])

iv/ Procedure based on isotonic regression (Mukerjee[1981], Charamza[1990])

v/ Blum procedure (Blum [1954])

vi/ Bather procedure (Bather [1990])

C/Problem of finding the root of the regression location parameter function

Let us denote by F_x the distribution function of the observations provided at the point x . Let $H: (R^2, B^2) \rightarrow (R, B)$. By the regression location parameter function we shall mean the real measurable function $m(x)$ which is defined as

$$m(x) \stackrel{\text{def}}{=} \underset{\theta}{\operatorname{argmin}} \int H(y, \theta) dF_x(y). \quad /5/$$

The regression function or the regression quantile function are the special cases of /5/ when choose $H(y, \theta) = (y - \theta)^2$ or $H(y, \theta) = |y - \theta|$. The methods listed below are proved to solve iteratively the equation

$$m(x) = \alpha. \quad /6/$$

i/ Procedure based on isotonic regression (Charamza [1990])

ii/ Procedure based on quasiisotonic regression (Charamza [1990])

D/Estimation of extreme of regression function

The methods from this paragraph are designed for finding the extreme of the function $m(x) \stackrel{\text{def}}{=} E g(x, \cdot)$, where the function g is the same as in /1/. The problem can be written mathematically as

$$m(x) \rightarrow \min! \quad /7/$$

i/ Kiefer-Wolfowitz method (Kiefer-Wolfowitz [1952])

ii/ Isotonic regression for differences (Charamza [1984])

iii/ Quasiisotonic regression for differences (Charamza [1990])

E/Estimation of quantile

The procedures in this paragraph can find the γ -quantile of the unknown distribution giving the random sample from it. All the methods are recursive.

i/Tierney method (Tierney [1981])

ii/ Procedure based on quasiisotonic regression (Charamza [1990])

iii/ Procedure based on isotonic regression (Charamza [1990])

iv/ Derman procedure (Derman [1957])

Let we have the unknown distribution function F . The problem is to find the γ -quantile of this distribution. In comparison with the problem formulated in the previous paragraph we do not have the random sample from this distribution for our dispose. The only information that we can obtain about this distribution is given by independent random variables $Y(X(t))$ distributed according the alternative law:

$$Y(X(t))=0 \quad \text{with probability } 1-F(X(t))$$

$$Y(X(t))=1 \quad \text{with probability } F(X(t)).$$

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i/ Procedure based on the isotonic regression (Charamza [1990])

ii/ Derman procedure (Derman [1957])

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If you are interested in obtaining this program, please contact author.

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